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Online pricing for multi-type of items

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ABSTRACT

This paper studies the online pricing problem in which there is a sequence of users who want to buy items from one seller. The single seller has k types of items and each type has limited copies. These users are arriving one by one at different times and are single-minded. When arriving, each user would announce her interested bundle of items and a non-increasing acceptable price function, which specifies how much she is willing to pay for a certain number of bundles. Upon the arrival of a user, the seller needs to determine immediately the number of bundles to be sold to the user and the price he would charge her. His goal is to maximize the sum of money received from users.

When items are indivisible, we show that no deterministic algorithm for the problem could have competitive ratio better than $O(hk)$ where h is the highest unit price that at least some user is willing to pay. Thus we focus on randomized algorithms. We derive a lower bound $\Omega(\log h + \sqrt{k})$ on the competitive ratio of any randomized algorithm for solving this problem. Then we give the first competitive randomized algorithm $\mathcal{R}p$-MULTI which is $O(\psi(h)\sqrt{k}\frac{\Delta}{\delta})$-competitive, where Δ and δ are respectively the maximum and minimum copies a type of items can have and $\psi(h)$ is a function growing slightly faster than $\log h$. When h is known ahead of time, the ratio is decreased to $O((\log h)\sqrt{k}\frac{\Delta}{\delta})$.

When items are divisible and can be sold fractionally, we concentrate on designing competitive deterministic algorithms. We give the first competitive deterministic algorithm $\mathcal{D}p$-MULTI which is $O(\psi(h)\sqrt{k}\frac{\Delta}{\delta})$-competitive. When h is known, the ratio can be decreased to $O((\log h)\sqrt{k}\frac{\Delta}{\delta})$. We also study randomized algorithms for the problem.

1. Introduction

The pricing problem and its many variants have been studied extensively in recent years, and they have found important applications in computational economics [4,12,17,14,1,16]. This paper studies an online version of the problem, in which there is a sequence of users who want to buy items from one single seller. The seller has k types of items, and each type has limited copies. The users are arriving one by one at different times and they are *single-minded*, which means that each user is only interested in a particular bundle (set) of items. When a user arrives, she would announce her interested bundle and an *acceptable price* function, which specifies how much she is willing to pay for a certain number of bundles. Then, the seller needs to determine immediately the number of bundles to be sold to the user and the price he would charge her. His goal is to maximize the total revenue, which is the sum of money received from the users.

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Table 1
The upper and lower bounds on the competitive ratios.

		Indivisible items	Divisible items
Deterministic	h known & $\Delta = \delta$	–	$O((\log h)(\log k)\sqrt{k})$ [21]
			$O((\log h)\sqrt{k})$
		$\Omega(hk)$	$\Omega(\log h + \log k)$ [21]
Randomized	h unknown	$O(\psi(h)\sqrt{k}\frac{\Delta}{\delta})$	$O(\psi(h)\sqrt{k}\frac{\Delta}{\delta})$
		$\Omega(\log h + \sqrt{k})$	–
	h known	$O((\log h)\sqrt{k}\frac{\Delta}{\delta})$	$O((\log h)\sqrt{k}\frac{\Delta}{\delta})$

The online pricing problem was first studied by Zhang et al. [20]. They focused on the simplest version of the problem, in which there is only one type of items and the items are *divisible* (i.e., they can be sold fractionally). They also assumed that each user has a non-increasing acceptable price function. Their main result is a deterministic algorithm with competitive ratio $O(\log h)$ where h is the highest unit price that at least some user is willing to pay. In [22], Zhang et al. proved that any deterministic algorithm for their problem must have competitive ratio at least $\frac{1}{2}\lfloor \log_2 h \rfloor$. In [18], we studied a similar problem, in which there is still only one type of items for sale, but now the items are *indivisible* and thus cannot be sold fractionally. We showed that it is much harder to handle indivisible items: any deterministic algorithm for the indivisible case must have competitive ratio at least $\Omega(h)$. We also gave a randomized algorithm for this case with competitive ratio $O(\log h)$, and proved that no randomized algorithm can do asymptotically better.

In [21], Zhang et al. studied the online price problem with k types of divisible items for sale, with the additional assumptions that (i) the highest price h is known ahead of time, and (ii) the quantities of different types of items are the same. They showed that the competitive ratio of any deterministic algorithms for this problem is at least $\Omega(\log h + \log k)$. They also gave a deterministic algorithm which is $O((\log h)(\log k)\sqrt{k})$-competitive.

There is yet no published result for the online pricing problem studied in this paper, in which the k types of items are indivisible, h is not known ahead of time, and different types of items may have different quantities for sale.

Our results. We will prove in Section 3 that no deterministic algorithm for online pricing of indivisible items can have competitive ratio better than $\Omega(hk)$. Thus we focus on randomized algorithms.

First, we apply Yao's lemma [19] to prove a lower bound on the competitive ratio of any randomized algorithm for online pricing indivisible items. The difficulty is to find an appropriate distribution of inputs, and to prove that no deterministic algorithm can perform well against this distribution. In our proof, we describe how to generate a well-designed random sequence of users having the same bundle sizes and acceptable price functions. We then show that against this random sequence, an offline algorithm can always accept as many as $\sqrt{k}$ users while the expected number of users that any deterministic online algorithm can accept is only 2. Then by Yao's lemma a lower bound of $\Omega(\sqrt{k})$ follows. We will also derive a lower bound of $\Omega(\log h)$ by a different method. Consequently, we conclude that any randomized algorithm for the problem is at least $\Omega(\log h + \sqrt{k})$-competitive. We note that even if the quantities of all types of items are the same, our lower bound still holds.

Next, we design the randomized algorithm $\mathcal{R}p$-MULTI for our problem. The algorithm is surprisingly simple and uses very little power of randomization; when it starts, it picks randomly a unit price τ and a bundle size threshold γ according to some fixed probability distributions. Whenever a user arrives and has a bundle size no less than the threshold γ, it simply tries to sell her the largest number of bundles that she is willing to buy under this unit price τ. Although $\mathcal{R}p$-MULTI is simple, its competitive analysis is not easy. By using a carefully designed probability distribution, we prove that the algorithm achieves a competitive ratio of $O(\psi(h)\sqrt{k}\frac{\Delta}{\delta})$, where Δ and δ are respectively the maximum and minimum copies a type of items can have and $\psi(h)$ is a function growing slightly faster than $\log h$ (in fact, $\psi(h) = o((\log h)^{1+\epsilon})$ for any $\epsilon > 0$). If h is known ahead of time, we can use a uniform probability distribution and reduce the competitive ratio to $O((\log h)\sqrt{k}\frac{\Delta}{\delta})$.

Finally, to make our study complete, we also consider online pricing for divisible items. We show that after a simple adaptation, $\mathcal{R}p$-MULTI can be used to solve the problem of online pricing of divisible items, and the competitive ratio is still $O(\psi(h)\sqrt{k}\frac{\Delta}{\delta})$, and when h is known ahead of time, the competitive ratio can be reduced to $O((\log h)\sqrt{k}\frac{\Delta}{\delta})$. We also design a deterministic algorithm $\mathcal{D}p$-MULTI for the divisible case, and it achieves the same asymptotic competitive ratios as $\mathcal{R}p$-MULTI. Recall that Zhang et al. [21] gave an $O((\log h)(\log k)\sqrt{k})$-competitive deterministic algorithm for the divisible items case with the assumptions that the quantities of all types are the same and h is known ahead of time. Compared with their results, $\mathcal{D}p$-MULTI is substantially better.

We summarize our results in Table 1.

Related works. There have been extensive studies on the offline pricing problem [14,6,8,7,2]. Guruswami et al. [14] studied the problem with unlimited supplies of m different types of items and n *single-minded* users. They gave an approximation algorithm which achieves expected revenue within an $O(\log n + \log m)$ factor of the total social welfare. This approximation ratio of $O(\log n + \log m)$ was proved to be tight by Briest [6] and Chalermsook et al. [8]. When m different types with unlimited supplies are for sale, and users are single-minded and each wants at most k types of items, Briest et al. [7] gave