Theoretical Computer Science 608 (2015) 166–177

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Conflict graphs and the SINR-capacity of the mean power scheme

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ARTICLE INFO

Article history:
Received 4 March 2014
Accepted 20 January 2015
Available online 22 January 2015

Keywords:
Wireless networks
SINR
Capacity
Conflict graphs

ABSTRACT

The Capacity problem in wireless networks is to select a maximum cardinality *feasible* subset of a set of transmission requests or *links*, where a set of links is feasible if the corresponding transmissions can be done simultaneously without collisions. We consider two models of feasibility: the SINR (Signal to Interference and Noise Ratio) model and a special *conflict graph* model. We show that if a special power assignment method is used (called the mean power scheme), then the solutions of the Capacity problem in the two models differ by at most a constant factor for any set of links in a doubling metric space of small dimension. This is not true for other power assignment methods in general. This result, besides showing that the mean power scheme is scalable between different models (as opposed to other power schemes), also has the potential to yield new results in the SINR model, using tools from graph theory, which we demonstrate in several examples.

1. Introduction

Capacity and Scheduling are fundamental problems in wireless networks. The Capacity problem is to select a maximum cardinality "feasible" subset of a given set of wireless links (transmission requests), where a set of links is feasible if they can transmit in the same time slot without collisions. The Scheduling problem is to split the given set of links to the minimum number of feasible subsets, so that each such subset can transmit in a separate time slot, and the number of slots required is minimum. In order to model feasibility, one must model interference in the wireless network. A common way of modeling interference in higher level analysis is by graph-based models. Here communication is modeled using a *conflict graph*, where communications corresponding to adjacent links cannot be in the same feasible set. In recent years considerable research has been conducted on the SINR (Signal to Interference and Noise Ratio) model, where the *cumulative* interference is considered, as opposed to the "binary" interference of graph-based models. It has been argued that graph based models can be too optimistic in not taking into account the cumulative interference. However, these models also have advantages such as simplicity. In fact, in networks with many obstacles and non-regular boundaries, the general SINR model seems to be hardly tractable, and graph-based models can be a reasonable option. Hence, it is interesting to find out how well do graph based models approximate the SINR model. We consider a special graph-based model (we call it simply "Conflict Graph Model") that is an approximation of the SINR model, and compare it with the SINR model with respect to the Capacity problem. The results we obtain in this context also give rise to some interesting corollaries for the Scheduling and Capacity problems in the SINR model.

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[1] Part of this work has been done while the author was a student at the University of Geneva, Switzerland.

http://dx.doi.org/10.1016/j.tcs.2015.01.031

Related work. The drawbacks of (disk) graph models have been demonstrated both theoretically and experimentally [1–4]. These drawbacks mainly stem from the fact that these models do not take the cumulative effect of interference into the account. On the other hand, there have also been attempts to use an underlying graph structure to obtain results for the SINR model [5] and to establish connections between the graphs and the SINR model [6,7]. It is known that certain graph-based models approximate SINR-Scheduling in doubling metric spaces with small dimension within a constant approximation factor when the lengths of the links are "almost equal" [8]. When the link lengths are allowed to be arbitrary, the degree of approximation depends on transmission power levels used by transmitters [9]. In particular, when the *linear or uniform* power assignment methods (or power schemes, as they are usually called) are used, the difference of the solution of the Scheduling problem in a special conflict graph model and in the SINR model is a factor in $O(\log\Delta)$ for any set of links in a doubling metric space with small dimension, but with an appropriately chosen power scheme (*mean power scheme*) the factor can be reduced to $O(\min\{\log\Delta, \log n\})$, where Δ is the ratio between the longest and shortest link lengths and n is the number of links [9]. These approximation bounds are easily carried over the Capacity problem.

The Capacity and Scheduling problems have been studied extensively during recent years. There are (centralized) constant factor approximation algorithms for the Capacity problem both for a variety of fixed power assignments [10] as well as for the *power control* variant, where the power assignment may also be optimized to yield better solutions for the problem [11]. These algorithms yield $O(\log n)$-approximation to the Scheduling problem. There are also randomized distributed $O(\log n)$-approximation algorithms for the Scheduling problem [12,13], however, the only constant factor approximation known for non-constrained link lengths is for the linear power scheme [14,15].

Our contribution. Our main result is that the optimum SINR-capacity achieved with the mean power scheme is approximated by the corresponding Conflict Graph-capacity within a constant factor, while for the uniform and linear power schemes the approximation factor can still be as large as $\Theta(n)$. This result has several implications for the SINR model. In particular, it is used to show that the mean power scheme is always asymptotically better than the uniform power scheme in terms of scheduling. The advantage of the mean power scheme has been known before for several network examples (e.g. [16]), and in terms of the Capacity problem [9], and we prove it in terms of the scheduling problem. We also use the graph-SINR relation to obtain another proof for the main result of [17] in a more restricted setting. Namely, we show that the mean power scheme is an $O(\log\log\Delta)$-approximation to the best possible power assignment in terms of SINR-Capacity. These results are obtained under the assumption that the links are located in a *doubling* metric space.

2. Models and definitions

2.1. Capacity and Scheduling problems

Let $\Gamma = \{1, 2, \ldots, n\}$ denote a set of n wireless links (transmission requests between a sender and a receiver node) in a metric space with distance function d, where the sender node of each link is assigned a certain power level. Using a specific communication model, one can define a *feasible set of links*, i.e. a set of links that can transmit with the given power levels in parallel, without collisions. The *Capacity problem* is to select a maximum cardinality feasible subset of Γ. The size of a maximum cardinality feasible subset is called the *capacity* of Γ. The *Scheduling problem* is to split Γ into the minimum number of feasible subsets. Each partition of Γ into feasible subsets is called *a schedule*, and the number of subsets (*slots*) is called the *length* of the schedule. Of course, the sizes of feasible sets and schedules depend on the power assignment of the nodes and the communication model. Below we formally define the communication models that we deal with.

2.2. The path-loss model of signal decay and cumulative interference

Let the sender node of each link i be assigned a transmission power $P(i) > 0$. A given transmission is normally exposed to the effect of the interference from other transmissions and the *ambient noise*. According to the path-loss propagation model [18], the received signal of each link i has power[2] $P_i = P(i)/l_i^\alpha$, where $l_i = d(s_i, r_i)$ is the distance between the sender and receiver nodes of link i and $\alpha > 0$ is the *path-loss exponent* (in practice, typically, $2 < \alpha < 6$ [18]). Similarly, the interference caused by link j at the receiver of link i has power $I_{ji} = P(j)/d_{ji}^\alpha$, where $d_{ji} = d(s_j, r_i)$ denotes the distance from the sender node of link j to the receiver node of link i. According to the SINR model, the transmission of a link i is successful if and only if the ratio of the received signal power over the total interference and the ambient noise is greater than a certain threshold $\beta \geq 1$:

$$\frac{P_i}{\sum_{j\in S\setminus\{i\}} I_{ji} + N} \geq \beta, \tag{1}$$

where the constant $N \geq 0$ denotes the noise and S is the set of links transmitting in the same time slot as i.

In this model, a subset S of Γ is feasible if (1) holds for each link $i \in S$.

We denote the *optimum SINR-capacity* of a set Γ with respect to a power assignment P as $OPTC_P(\Gamma)$, and the *optimum SINR-schedule length* by $OPTS_P(\Gamma)$.

[2] More precisely, this is the average value of the received signal power. Even though we use static values of the power in our calculations, it can be seen that the result will not be much different from the results obtained e.g. in the statistical Rayleigh fading model [19].