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Combinatorial auctions with verification are tractable ☆

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A B S T R A C T

We study mechanism design for social welfare maximization in combinatorial auctions with general bidders given by demand oracles. It is a major open problem in this setting to design a deterministic truthful auction which would provide the best possible approximation guarantee in polynomial time, even if bidders are double-minded (i.e., they assign positive value to only two sets in their demand collection). On the other hand, there are known such randomized truthful auctions in this setting. In the general model of verification (i.e., some kind of overbidding can be detected) we provide the first *deterministic* truthful auctions which indeed provide essentially the best possible approximation guarantees achievable by any polynomial-time algorithm even if the complete input data is known. This shows that deterministic truthful auctions have the same power as randomized ones if the bidders withdraw from unrealistic lies. Our truthful auctions are based on greedy algorithms and our approximation guarantee analyses employ linear programming duality based techniques. Finally, our truthfulness analyses are based on applications of the cycle-monotonicity technique which we show to surprisingly couple with the greedy approach.

1. Introduction

Algorithmic Mechanism Design attempts to marry up computational and economic considerations. Indeed, a mechanism has to deal with the strategic behavior of the participants and still has to compute the outcome efficiently. Natural applications of interest are protocols over the Internet where participating (commercial) entities pursue their own objectives: Strategic and algorithmic issues have to be considered together [31].

Facing a *truthful* mechanism, participants are always rationally motivated to correctly report their private information. (For an introduction to the basics of Mechanism Design we refer to Chapter 9 in [32].) This setting (i.e., participants reporting information) and the focus on truthful mechanisms is, by the Revelation Principle, without loss of generality (see [27,31]). Many works in the literature (including this one) require truthtelling to be a dominant strategy equilibrium. This solution concept is very robust, but sometimes it may be too strong to *simultaneously* guarantee truthfulness and computational efficiency.

This is the case for the arguably main technique known in the field: VCG mechanisms [37,11,20]. VCG mechanisms are truthful once the exact optimal outcome is computed. This clashes with computational aspects. In fact, for many interesting

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applications exact optimization is an NP-hard problem. So, we have to content ourselves with efficient approximation algorithms. Unfortunately, VCG mechanisms fail if the output solution is only approximately optimal, and thus they cannot be applied in these cases [30]. The main challenge in Algorithmic Mechanism Design is to go beyond VCG and design efficient truthful mechanisms for those hard applications.

The design of truthful *Combinatorial Auctions* (CAs, see Section 2 for definition) is the canonical problem in the area suffering from this drawback of VCG mechanisms. In a combinatorial auction we have a set U of m goods and n bidders. Each bidder i has a *private* valuation function v_i that maps subsets of goods to nonnegative real numbers ($v_i(\emptyset)$ is normalized to be 0). Agents' valuations are monotone, i.e., for $S \supseteq T$ we have $v_i(S) \geq v_i(T)$. Notice that the number of these valuations is exponential in m while we need mechanisms running in time polynomial in m and n. So, we have to assume how these valuations are encoded. As in, e.g., [5,15,12], we assume that the valuations are represented as black boxes which can answer a specific natural type of queries, called *demand queries*.[1] The goal is to find a partition $S_1, \ldots, S_n$ of U such that $\sum_{i=1}^n v_i(S_i)$ – the *social welfare* – is maximized. CAs can be strategically solved by means of a VCG mechanism. But the computational optimization problem is NP-hard to solve optimally or even to approximate: neither an approximation ratio of $m^{\frac{1}{2}-\epsilon}$, for any constant $\epsilon > 0$, nor of $O(\frac{d}{\log d})$ can be obtained in polynomial time [29,26,21], where m is the number of goods to sell and d denotes the maximum size of subsets (bundles) of goods bidders are interested in (see Section 2 for a formal definition). Therefore VCG mechanisms cannot be used to solve CAs efficiently *and* strategically. To date we do not have yet a complete picture of the hardness of CAs. That is, the question in Chapter 12 of [32], still remains unanswered:

> "What are the limitations of deterministic truthful CAs? Do approximation and dominant-strategies clash in some fundamental and well-defined way for CAs?"

1.1. Related work and our contributions

In the attempt to give answers to the questions above, a large body of literature has focused on the design of efficient truthful CAs under different type of assumptions. The first results of tractability rely on the *restriction of the bidders' domains*. If we restrict bidders to be interested in only single set, the so-called *single-minded* domain, CAs are very well understood: a certain monotonicity property is sufficient for truthfulness, can be guaranteed efficiently and leads to the best approximation ratio (in terms of m) possible[2] [26]. For single-minded domains, a host of other truthful CAs have been found (see, e.g., [1,28,16,6]). Furthermore, a number of truthful CAs have been provided under different assumptions (i.e., restriction) on the valuation domains (see Figure 11.2 in [32] for a complete picture).

The situation is very different for the multi-dimensional domains where bidders can evaluate different sets of goods differently. Very few results are known and they still do not answer the questions above. In [22] an algorithm that optimizes over a carefully chosen range of solutions (i.e., a *maximal-in-range (MIR)* algorithm) is coupled with VCG payments and shown to be a truthful $O(m/\sqrt{\log m})$ approximation. A second result is the mechanism in [4] that applies only to the special case of auctions with many duplicates of each good. No other *deterministic* positive results are known even for the simplest case of double-minded bidders (i.e., bidders with *only* two non-zero valuations). On the contrary, we know that VCG-based mechanisms do have limitations for CAs [14] and that obtaining efficient truthful CAs is not tractable when using deterministic MIR algorithms [8]. Positive results are instead known for *randomized* truthful CAs. In particular, in [15] the authors show a universally truthful CA which gives an $O(\sqrt{m})$-approximate solution with all but constant probability. The approach has been later extended in [12] to guarantee the same approximation ratio while reducing the error probability to $O(\log m/\sqrt{m})$. Due to this error probability, these solutions do not *guarantee* the approximation ratio. The success probability cannot be amplified either: Repeating the auction would destroy truthfulness [12]. An $O(\sqrt{m})$-approximate truthful in expectation mechanism is given in [25].[3]

The power of randomization for CAs (and, in general, for mechanism design) does not seem to be an accident as shown by the randomized truthful in expectation FPTAS for multi-unit auctions [13]. However, randomization comes with its tolls: approximation might not be guaranteed [12] and/or assumptions on the risk attitude of bidders have to be made [25]. The challenge is therefore to reconcile incentives and efficient *deterministic* computation.

In this paper, we show deterministic truthful CAs with multi-dimensional bidders that run in polynomial time and return (essentially) best possible approximate solutions under a general and well motivated assumption. We use an approach which is orthogonal to the restriction of the domain used to obtain previously known tractable CAs. We keep the most general valuation domains but we *restrict the way bidders can lie*, an assumption well motivated in economics [17,18]. Specifically, we introduce the idea of *verification* [31] to the realm of CAs.

Verification and CAs: motivation It was observed in [17] that in the economic scenario of "regulation of a monopolist" it makes sense to assume that no bidder will *ever* overbid: overbidding can be sometimes infinitely costly. One of such cases

[1] In a demand query (with bundle prices) the bidder is presented with a compact representation of prices $p(S)$ for each $S \subseteq$ U, and the answer is the bundle S that maximizes the profit $v(S) - p(S)$.

[2] The best here refers to any (even not truthful) polynomial-time algorithm.

[3] For the auctions in [12,25] no approximation guarantee in terms of d is claimed.