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Termination of the iterated strong-factor operator on multipartite graphs ☆

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A B S T R A C T

The clean-factor operator is a multipartite graph operator that has been introduced in the context of complex network modelling. Here, we consider a less constrained variation of the clean-factor operator, named strong-factor operator, and we prove that, as for the clean-factor operator, the iteration of the strong-factor operator always terminates, independently of the graph given as input. Obtaining termination for all graphs using minimal constraints on the definition of the operator is crucial for the modelling purposes for which the clean-factor operator has been introduced. Moreover we show that the relaxation of constraints we operate not only preserves termination but also preserves the termination time, in the sense that the strong-factor series always terminates before the clean-factor series. In addition to those results, we answer an open question from Latapy et al. [12] by showing that the iteration of the factor operator, which is a proper relaxation of both operators mentioned above, does not always terminate.

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1. Introduction

One of the main challenges in modelling real-world complex networks (like internet topology, web graphs, social networks, or biological networks) is to design general models able to reproduce both the heterogeneous degree distribution of these networks and their high local density (clustering coefficient). One of the most promising approaches to do so is the one proposed by Guillaume and Latapy [7,8], which aims at generating synthetic complex networks by generating their maximal cliques rather than their edges. The main difficulty in this approach is to reproduce correctly the overlaps of the maximal cliques of the graph, which is prevalent in practice. To that purpose, Latapy et al. [12] proposes to encode the non-trivial overlaps of the maximal cliques of a graph G by a multipartite graph which is defined by iteratively applying a multipartite-graph operator, named the *weak-factor graph*, starting from the vertex-clique-incidence bipartite graph of G (see Definition 4 below and example in Fig. 1). Unfortunately, the most natural definition of this operator gives series that do not terminate for some graphs G. In these cases, the object on which is based the random generation process of the model is undefined. In order to solve this issue, Latapy et al. [12] designed a variation of the weak-factor operator, called

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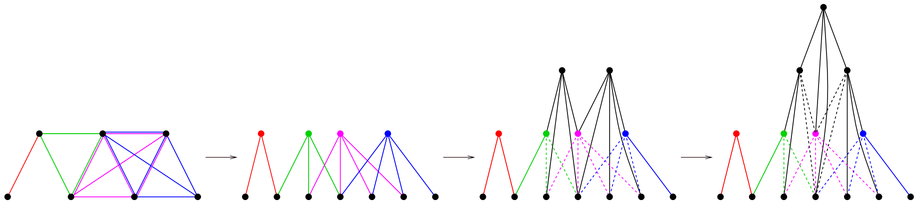

Fig. 1. Example of the weak-factor series of some graph G. From left to right: the original graph $G = G_0$, its vertex-clique-incidence bipartite graph G_1, the tripartite graph G_2 of the weak-factor series of G, and the quadripartite graph G_3 of the series. In this case, the weak-factor series terminates as the factorisation of G_3 is not effective (see Definition 1). The dashed edges are those belonging to some non-trivial maximal bicliques used in the factorisation steps.

the clean-factor, such that the corresponding series terminates for all graphs. The idea of this variation is to add some constraints to the factorising step defining the operator (see Definition 1 below) in order to force termination of the series and still capture the overlapping structure of the maximal cliques of the graph. But it turns out that the constraints added to the operator to obtain termination make the generation process of the model much more difficult to design. Therefore, for modelling purposes, it is crucial to guarantee termination for all graphs by imposing constraints as light as possible. We believe that this question of finding the minimal constraints that guarantee termination of the series is also of great theoretical interest.

1.1. Our contribution

Our main contribution is to design a relaxation of the clean-factor operator [12], called the strong-factor operator, which is much less constrained and for which we prove that the corresponding series also terminates for all graphs. Namely, we replace the condition requiring equality of the neighbourhoods of vertices in the definition of the clean-factor operator by a condition requiring only that these vertices share at least two neighbours in common, which constitutes a strong relaxation of the previous definition. In addition, we show that this relaxation not only preserves termination but also does not delay it: the strong-factor series, though less constrained, always terminates before the clean-factor series. For sake of completeness, let us mention that in [4], which is a complete and improved version of Latapy et al. [12], the constraints in the definition of the clean-factor operator are slightly different and are expressed in a weaker way. But these weaker constraints actually imply that the constraints of the definition used in [12] are also satisfied. Therefore, the strong-factor operator introduced here is a proper relaxation of both versions [12,4] of the clean-factor operator.

Besides the results we obtain on the termination of the strong-factor series, we also provide a complete characterisation of the levels of the series, in terms of intervals of a poset, that is worth of interest in itself. This characterisation is very simple and gives an insight on the structure of the clean-factor series that, we believe, may also be useful to prove termination or non-termination of other multipartite graph operators. In addition, it provides an efficient way to compute the strong-factor series, by avoiding the computation of maximal bicliques.

Finally, we answer an open question of Latapy et al. [12] by showing that the factor series, which is a relaxation of both the clean-factor series and the strong-factor series, does not terminate for some graphs.

1.2. Related work

The strong-factor operator which we study here is a variation of the weak-factor operator, which operates on multipartite graphs and which is defined using the bicliques between the upper level and the rest of the multipartite graph. For graphs, closely related operators have been defined using the cliques or the bicliques of the graph, and many works addressed the question of convergence of the series obtained by iteratively applying these operators to an input graph. There exist several definitions of convergence in the literature. The notion of termination we use here for the multipartite graph series we consider is somehow equivalent to the convergence notion used in [1] in the context of graph series, and is a particular case of convergence of the definition used in [14].

For the well-known clique graph operator (see [15] for a survey) the question of convergence has received a lot of attention [14,1]. Most of the efforts focussed on obtaining convergence results, or divergence results, for some particular graphs or graph classes [9–11,13]. Similar questions have been addressed recently for the biclique graph operator [5,6], which also operates on graphs but using bicliques instead of cliques. Let us mention, that another closely related graph operator called edge-clique-graph operator has been studied (see e.g. [3,2]) but, to the best of our knowledge, the question of the convergence of its iterated series has not been investigated.

It must be clear that none of these three operators, clique graphs, biclique graphs and edge-clique graphs, which are defined on graphs, is equivalent to one of the multipartite-graph operators we consider here. And the convergence or divergence results obtained previously for these graph operators do not imply the non-termination and termination results we prove here for the factor graph and the strong-factor graph respectively.

Moreover, it is worth noticing that, even though it deals with a notion of convergence, the question we address in this paper is orthogonal, and complementary, to the one addressed in all the previously cited works. Indeed, we do not intend

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