Theoretical Computer Science 591 (2015) 28–59

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

The searchlight problem for road networks

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ARTICLE INFO

Article history:
Received 20 May 2013
Received in revised form 21 April 2015
Accepted 23 April 2015
Available online 5 May 2015
Communicated by D. Peleg

Keywords:
The searchlight problem
Graph searching
Lines
Line segments
Grids

ABSTRACT

We consider the problem of searching for a mobile intruder hiding in a road network given as the union of two or more lines, or two or more line segments, in the plane. Some of the intersections of the road network are occupied by stationary guards equipped with a number of searchlights, each of which can emit a single ray of light in any direction along the lines (or line segments) it is on. The goal is to detect the intruder, that is, to illuminate its location. Guards may alter the direction in which they aim a searchlight, but need to switch it off for some finite time interval to effect the change. In contrast, the intruder may move with arbitrary speed along the network (but cannot pass guards) and exploit this time interval to recontaminate previously illuminated sections of the network. For various classes of road networks characterized by the number n of lines (or line segments) comprising it and the number g $(\leq n-1)$ of possible locations of guards (fixed in advance and guaranteed to give complete coverage), we present several upper and lower bounds on the worst-case number of searchlights, each placed at one of the guard positions, required to successfully search a given road network. In particular, we prove the following results:

1. $\min\{2g-1, n-2\}$ searchlights are sometimes necessary and $\min\{\frac{7}{3}g, n\}-1$ are always sufficient for searching a road network given as the union of n lines;
2. $\Omega(g \cdot \log \frac{n}{g})$ searchlights are sometimes necessary and $O(g^2 \cdot \log n)$ searchlights are always sufficient for searching a road network given as the union of n line segments, and
3. at most one searchlight per guard position, and hence a total of at most g searchlights, is always sufficient for searching a road network given as the union of axis-aligned lines or line segments.

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[1] Partially supported by MNiSW Grant No. N206 379337 (2009–2011).
[2] Supported in part by UWM Research Growth Initiative.
[3] Partially supported by MNiSW Grant No. N516 196437 (2009–2012).

http://dx.doi.org/10.1016/j.tcs.2015.04.026

The proofs of the upper bounds induce algorithms for generating a search schedule for detecting the intruder using the claimed number of searchlights.

1. Introduction

Assume that a mobile intruder capable of moving continuously and arbitrarily fast is hiding in the streets of a dark city, where each street is modeled as a line or a line segment in the plane. Suppose that a number of stationary guards have been placed on the streets, each equipped with one or more searchlights, where a searchlight can be aimed in any street direction (from the guard position) and emit a single ray of light in that direction. The objective of the guards is to detect the intruder using the searchlights, where the intruder is considered *detected* at the moment he is illuminated by one of the searchlights or he reaches a position where a guard is located.

Formally, for $n \geq 2$, let $\mathcal{L} = \{L_1, L_2, \ldots, L_n\}$ be a set of n distinct lines, or n distinct (open or closed) line segments, in the plane, such that their union $\bar{\mathcal{L}} = L_1 \cup L_2 \cup \cdots \cup L_n$ is connected. In the former case we assume that no two collinear line segments in $\mathcal{L}$ intersect each other. For now, we continue the discussion assuming the L_i's are lines. All definitions and assumptions naturally carry over to the case of line segments without any change. We call $\mathcal{L}$ a *road network*. A point p is *visible* from a point q in $\bar{\mathcal{L}}$ if p and q lie on a common line in $\mathcal{L}$. A finite set of points $V \subseteq \bar{\mathcal{L}}$ is a *guard set* of $\mathcal{L}$ if every point in $\bar{\mathcal{L}}$ is visible from at least one point in V. The points in V are called *guards*. Hereafter we assume guards are distinct points and are always placed at intersections of lines in $\mathcal{L}$, since any guard set can be transformed into another without increasing the size by relocating every guard lying on a single line to a nearest intersection. The size γ of a smallest guard set of $\mathcal{L}$ is called the *guard number* of $\mathcal{L}$. Obviously, $1 \leq \gamma \leq n-1$ since $\bar{\mathcal{L}}$ is connected. Given a guard set V of $\mathcal{L}$, where $\gamma \leq |V| = g \leq n-1$, we call the pair $\mathcal{A} = (\mathcal{L}, V)$ an (n, g)-*arrangement*, or simply, an *arrangement*.

A guard $p \in V$, placed at an intersection of k lines, can have one or more *searchlight* that can emit a single ray of "light" that can be aimed in the direction of any of the $2k$ half-lines that meet at p and "illuminate" the points on it.[4] Each searchlight can be aimed in only one direction at a time, and we assume that the direction of the searchlight cannot be changed instantaneously. One way to interpret this assumption is that F has to be "turned off" while it changes directions. See Fig. 1 for an illustration of this assumption. In the following we often use the term "rotate" to refer to the action of changing direction of a searchlight.

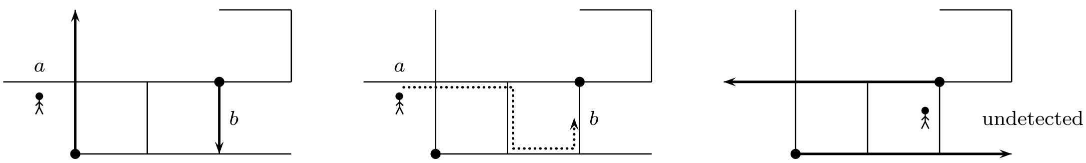

Fig. 1. Two searchlights shown on the left are rotated simultaneously as shown on the right. As is shown in the middle, while the searchlights are "turned off" during the rotation, an intruder hiding in a can move to b without being detected and "recontaminate" b.

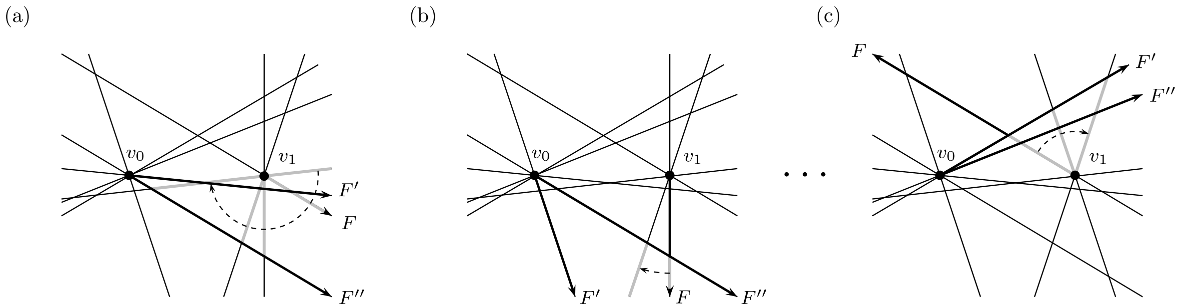

Fig. 2. A sample search strategy for an (8, 2)-arrangement. The gray line components searched in (a), together with the half-line illuminated with F', are not recontaminated during the rotation of F', due to the choice of the first wedge and the assumption that the intruder cannot pass through v_0 and v_1 without being detected.

A *searching strategy* for $\mathcal{A} = (\mathcal{L}, V)$ using a given set of searchlights placed at guard positions in V specifies, as a function of time, the directions in which the searchlights are aimed. A searching strategy *successfully searches* (or *clears*) $\mathcal{L}$ if it is not

4 If $\mathcal{L}$ consists of line *segments*, then there will be *up to* $2k$ possible directions of the searchlight.