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Confident and consistent partial learning of recursive functions ☆

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ABSTRACT

Partial learning is a criterion where the learner infinitely often outputs one correct conjecture while every other hypothesis is issued only finitely often. This paper addresses two variants of partial learning in the setting of inductive inference of functions: first, confident partial learning requires that the learner also on those functions which it does not learn, singles out exactly one hypothesis which is output infinitely often; second, essentially class-consistent partial learning is partial learning with the additional constraint that on the functions to be learnt, almost all hypotheses issued are consistent with all the data seen so far. The results of the present work are that confident partial learning is more general than explanatory learning, incomparable with behaviourally correct learning and closed under union; essentially class-consistent partial learning is more general than behaviourally correct learning and incomparable with confident partial learning. Furthermore, it is investigated which oracles permit to learn all recursive functions under these criteria: for confident partial learning, some non-high oracles are omniscient; for essentially class-consistent partial learning, all PA-complete and all oracles of hyperimmune Turing degree are omniscient.

1. Introduction

Gold [8] initiated the study of inductive inference, which investigates various forms of learning recursive functions and r.e. sets in the limit. Gold originally considered recursive learners which receive piecewise information about the graph of an unknown recursive function, presented in the natural ordering of the input values, while they conjecture a sequence of hypotheses which syntactically converges to a correct conjecture. Osherson, Stob and Weinstein [16] generalised Gold's paradigm to *partial learning* by weakening the convergence requirement in such a way that one correct hypothesis is required to be conjectured infinitely often while every other hypothesis is conjectured only finitely often.

On one hand, many natural examples of classes of recursive functions fail to be identifiable in the limit by any recursive learner, even in the broadest sense of semantic convergence [1]; this deficiency has motivated alternative approaches to learnability in the inductive inference such as the above mentioned one of partial learning. Feldman [6], for example, showed that a decidable rewriting system (drs) is always learnable from positive information sequences in a certain restricted sense. When introducing their criterion, Osherson, Stob and Weinstein [16] discovered that the whole class *REC* of recursive functions is partially learnable and that partial learnability is much more general even than behaviourally correct

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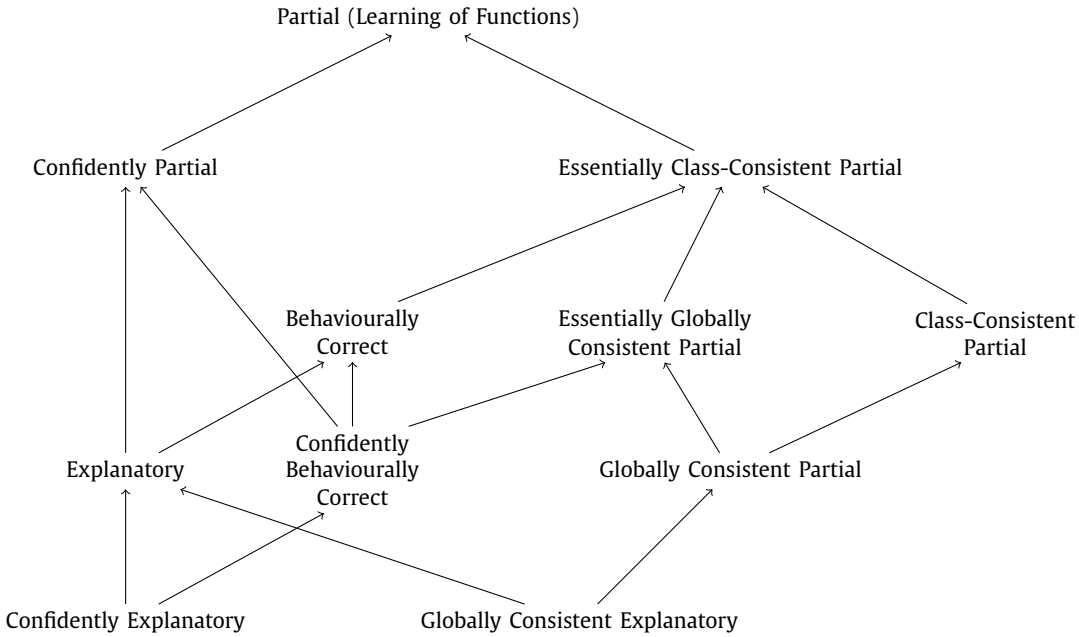
learnability. Subsequently, researchers thought that partial learning is too general and studied what happens when partial learning is combined with more restrictive constraints, most notably consistency which was introduced by Bārzdiņš [1] and which means that each hypothesis e for some data $f(0), f(1), \ldots, f(n)$ satisfies that $\varphi_e(m)$ is defined and equal to m for all $m \leq n$. Indeed, consistent partial learners can easily be shown to fail learning the class of all recursive functions. Wiehagen and Zeugmann [18] and later Grieser [9] and Jain and Stephan [13] studied consistent learning and partial consistent learning. Other constraints of partial learning were neglected, mostly as the corresponding notions coincided with partial learning itself.

The present work wants to fill this gap; as a start, the notion of confident partial learning is brought over to function learning from the original setting of language learning for which it was introduced by Gao, Stephan, Wu and Yamamoto [7]. In addition the present work introduces the notions of essentially class-consistent and essentially globally consistent partial learning; these learning notions align as follows with other notions of inductive inference:



In the following, the concepts and results are explained in more detail. Confidence in partial learning enforces that the learner must issue exactly one hypothesis infinitely often when reading the data of one given object, even if this object does not belong to the target class. In the case of language learning, the notion turned out to be restrictive [7]: even the class of all cofinite sets is not confidently partially learnable.

On the other hand, confident partial learning has some regularity properties. In the here investigated case of function learning, one can show that the union of confidently partially learnable classes is confidently partially learnable (this is parallel to the corresponding result for confidently explanatory learning of classes of functions); furthermore, this notion is more general than Gold's original notion of explanatory learning [3,8] and incomparable to the more general notion of behaviourally correct learning [1]. Confidence, though restrictive, is nevertheless a desirable quality of a learner as the learner tries always to come up with a hypothesis, even in the case that the data is arbitrary. This property permits to prove some desirable aspects of confidently learnable classes, for example, that the union of two confidently learnable classes is again confidently learnable.

Consistency, whilst a fairly stringent learning constraint, may be quite a desirable quality of learners, especially when the inductive inference paradigm is viewed as a model for scientific discovery. It is conceivable that a scientific theory with any epistemic value must be developed in accordance with empirical data, and, while allowing for a certain margin of error due to experimental inaccuracies, should possess a set of potential falsifiers that determine the consistency or non-consistency of its fundamental assumptions under the conditions of a controlled experiment [15]. Briefly, the falsificationist methodological rule expounded by Popper [17] states that a scientific theory is to be rejected if it is inconsistent with some basic statement unanimously accepted by the scientific community. In view of this benchmark by which science progresses, one may argue that consistency with empirical data is an essential characteristic of the hypotheses issued by scientists modelled as recursive learners.

Jain and Stephan [13] showed that the class *REC* of all recursive functions can be consistently partially learnt relative to an oracle A iff A has hyperimmune degree. In the present paper, we show that by weakening this learning constraint to *essential consistency*, under which a recursive learner is only required to be consistent on cofinitely many segments of a sequence input, *REC* can be partially inferred relative to any PA-complete oracle. Thus, by the result of Jockusch and Soare