Theoretical Computer Science 582 (2015) 48–59

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Fault tolerance and diagnosability of burnt pancake networks under the comparison model

Sulin Song [a,c], Xiaoyan Li [b,c], Shuming Zhou [a,c,*], Mi Chen [c]

[a] Fujian Provincial Key Laboratory of Network Security and Cryptology, Fujian Normal University, Fuzhou, Fujian, 350007, PR China
[b] School of Mathematics, Ningde Normal University, Ningde, Fujian, 352100, PR China
[c] School of Mathematics and Computer Science, Fujian Normal University, Fuzhou, Fujian, 350007, PR China

ARTICLE INFO

Article history:
Received 10 August 2014
Received in revised form 31 December 2014
Accepted 16 March 2015
Available online 23 March 2015
Communicated by R. Klasing

Keywords:
Burnt pancake networks
Fault tolerance
Comparison model
Extra connectivity
Conditional diagnosability

ABSTRACT

Processor fault diagnosis plays an important role in measuring the reliability of a multiprocessor system, and the diagnosabilities of many well-known multiprocessor systems have been investigated. The conditional diagnosability has been widely accepted as a new measure of diagnosability by assuming an additional condition that any faulty set can't contain all the neighbors of any node in a multiprocessor system. In this paper, we explore algebraic and combinatorial properties of burnt pancake networks, and investigate the structural vulnerability as well as super and extra connectivities. Furthermore, we show that the classic diagnosability and the conditional diagnosability of n-dimensional burnt pancake network BP_n ($n \geq 4$) are n and $3n - 4$, respectively.

1. Introduction

Processors of a multiprocessor system are connected according to a given interconnection network. Fault tolerance is especially important for interconnection networks, since failures of network components are inevitable when the size of network grows rapidly. To be reliable, the rest of the network should stay connected when component faults occur. Obviously, this can only be guaranteed if the number of faults is smaller than the minimum degree of the network. When the number of faults is larger than the minimum degree, some extensions of connectivity are necessary, since the graph may become disconnected. Some generalizations of connectivity were introduced and examined for various classes of graphs [7], including *super connectedness* and *tightly super connectedness*, where only one singleton can appear in the remaining network. As the number of faults of the graph increases, it is desirable that when a few processors are separated from the rest, the largest component of the surviving network stays connected and the network will continue to be able to function. Many interconnection networks have been examined in this aspect, when the number of faults is roughly twice the minimum degree [8]. One can even go further and ask what happens when more vertices are deleted. This has been examined for the hypercube in [39–41] and for certain Cayley graphs generated by transpositions in [9], and it has been shown that the surviving network has a large component containing almost all vertices. Recently, Chang et al. [4,6] have explored the *extra-connectives* of hypercube as well as its variants, which will be useful in establishing the conditional diagnosability of the multiprocessor systems based on these structures.

* Corresponding author at: School of Mathematics and Computer Science, Fujian Normal University, Fuzhou, Fujian, 350007, PR China.
E-mail address: zhoushuming@fjnu.edu.cn (S. Zhou).

http://dx.doi.org/10.1016/j.tcs.2015.03.027

The process of identifying faulty processors in a system by analyzing the outcomes of available interprocessor tests is called *system-level diagnosis*. In 1967, Preparata et al. [34] established a foundation of system diagnosis and an original diagnostic model, called the *PMC model*. Its target is to identify the exact set of all faulty vertices before their repair or replacement. All tests are performed between two adjacent processors, and it was assumed that a test result is reliable (respectively, unreliable) if the processor that initiates the test is fault-free (respectively, faulty). The *comparison-based diagnosis models*, first proposed by Malek [33] and Chwa and Hakimi [12], have been considered to be a practical approach to fault diagnosis in the multiprocessor systems. In these models, the same job is assigned to a pair of processors in the system and their outputs are compared by a central observer. This central observer performs diagnosis using the outcomes of these comparisons. Maeng and Malek [32] extended Malek's comparison approach to allow the comparisons carried out by the processors themselves. Sengupta and Dahbura [35] developed this comparison approach such that the comparisons have no central unit involved.

The classical diagnosability of a system is quite small owing to the fact that it ignores the unlikelihood of some specific processors failing at the same time. Therefore, it is attractive to develop different measures of diagnosability based on application environment, network topology, and statistics related to fault patterns. Lai et al. [26] developed a new measure of diagnosability, named as *conditional diagnosability* of a system under the PMC model, which assumes it is impossible that all adjacent nodes of one node are faulty simultaneously. That is, conditional diagnosability is the diagnosability under the condition that all adjacent nodes of any node can't be faulty simultaneously. And they also showed that the conditional diagnosability of n-dimensions hypercube is $4(n-2)+1$ for $n \geq 5$. Lin et al. [31] determined that the conditional diagnosability of arrangement graph $A_{n,k}$ ($k \geq 2, n \geq k+2$) is $(4k-4)(n-k)-3$. Chang and Hsieh [5] explored the structural properties of star graph, which is isomorphic to $A_{n,n-1}$, and determined its conditional diagnosability. Chang et al. [3] also established that the conditional diagnosability of k-ary n-cubes is $8n-7$ for $k \geq 4$ and $n \geq 4$. Lin et al. [29] introduced the conditional diagnosis under the comparison model. By evaluating the size of connected components, they obtained that the conditional diagnosability of the star graph S_n under the comparison model is $3n-7$. Through the same method, Hsu et al. [19] proved that the conditional diagnosability of the hypercube Q_n is $3n-5$. Specifically, Cheng et al. [10] and Zhou [42] established the conditional diagnosability of (n,k)-star graphs, respectively; Zhou et al. [43] also established the conditional diagnosability of (n,k)-arrangement graphs under the comparison diagnosis model. For the n-dimensional shuffle-cube SQ_n, Lin et al. [30] proved that the conditional diagnosability is $3n-9$ ($n=4k+2$ and $k \geq 2$). This paper, through the analysis of fault tolerance, establishes the conditional diagnosability of BP_n under the comparison model and shows that it is $3n-4$ for $n \geq 4$.

The rest of this paper is organized as follows. Section 2 introduces terminologies and notations about pancake networks and burnt pancake networks. Section 3 is devoted to structural properties of burnt pancake networks, and shows that burnt pancake networks are Cayley graphs based on the wreath product of the cyclic group Z_2 and symmetric group S_n. The tightly super connectivity and extra connectivity of BP_n are also derived through its vulnerability analysis. Section 4 concentrates on the main result of the paper that the conditional diagnosability of BP_n ($n \geq 4$) under the comparison model is $3n-4$. Section 5 concludes the paper.

2. Preliminaries

2.1. Terminologies and notations

For notation and terminology not defined here we follow [38]. Specifically, we use a graph $G = G(V, E)$ to represent an interconnection network, where each node $u \in V$ denotes a processor and each edge $(u, v) \in E$ denotes a link between nodes u and v. If at least one end-vertex of an edge is faulty, the edge is said to be *faulty*; otherwise, the edge is said to be *fault-free*. For any vertex u of the graph $G = (V, E)$, the neighborhood $N_G(v)$ of vertex v in G is defined as the set of all vertices which are adjacent to v, i.e., $N_G(v) = \{u \in V \mid uv \in E\}$. Let S be a subset of V, the subgraph of G induced by S, denoted by $G[S]$, is the graph with the vertex-set S and the edge-set $\{(u, v) \mid (u, v) \in E, u, v \in S\}$. We define $N_G(S) = \{v \in V \setminus S \mid \exists u \in S, uv \in E\} = (\bigcup_{u \in S} N(u)) \setminus S$. We also denote $N_G[S] = N_G(S) \cup S$. When G is clear from the context, we use $N(v)$ to replace $N_G(v)$, $N(S)$ to replace $N_G(S)$. For brevity, $N(\{u, v\})$ and $N[\{u, v\}]$ are written as $N(u, v)$ and $N[u, v]$, respectively. We also denote, by $|N(u)|$, the degree $d(u)$ of u. Let $\delta(G) = min\{d(u) \mid u \in V\}$ be the minimum degree of G. The neighborhood of a set S in a subgraph U is defined as the set $N_U(S) = (\bigcup_{v \in S} N_U(v)) \setminus S$.

When G is a graph and any subset $F \subset V(G)$, the notation $G \setminus F$ denotes a graph obtained by removing all vertices in F from G and deleting those edges with at least one end-vertex in F, simultaneously. The notation $M \setminus N$ denotes the *difference set of two sets* M and N, i.e., $M \setminus N = \{u \mid u \in M, u \notin N\}$. We denote $M \triangle N$ as the *symmetric difference set of two sets* M and N, i.e., $M \triangle N = (M \setminus N) \cup (N \setminus M) = \{x \mid x \in M \cup N, x \notin M \cap N\}$. For any two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, we denote $G_1 \cup G_2$ (resp. $G_1 \cap G_2$) as a graph with vertex-set $V_1 \cup V_2$ (resp. $V_1 \cap V_2$) and edge-set $E_1 \cup E_2$ (resp. $E_1 \cap E_2$). A *path* in a graph is a sequence of distinct vertices so that there is an edge joining consecutive vertices, with the length being the number of vertices in the sequence minus 1. A *cycle* is a path of length at least 3 where there is an edge joining the first and last vertices. A path (or cycle) of length k is called a k-path (or k-cycle). We use $d(u, v)$ to denote the distance between u and v, the length of a shortest path between u and v in G.

If $G \setminus F$ is disconnected, F is called a *separating set*, namely *vertex-cut*. A separating set F is called a *k-separating set* if $|F| = k$. The maximal connected subgraphs of $G \setminus F$ are called *components*. A component is trivial if it has no edges;