Theoretical Computer Science 553 (2014) 57–63

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Wireless capacity with arbitrary gain matrix ☆

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ARTICLE INFO

Available online 9 October 2013

Keywords:
Wireless networks
SINR model
Semi-definite programming

ABSTRACT

Given a set of wireless links, a fundamental problem is to find the largest subset that can transmit simultaneously, within the SINR model of interference. Significant progress on this problem has been made in recent years. In this note, we study the problem in the setting where we are given a fixed set of arbitrary powers each sender must use, and an arbitrary gain matrix defining how signals fade. This variation of the problem appears immune to most algorithmic approaches studied in the literature. Indeed it is very hard to approximate since it generalizes the max independent set problem. Here, we propose a simple semi-definite programming approach to the problem that yields constant factor approximation, if the optimal solution is strictly larger than half of the input size.

1. Introduction

Given a set $L = \{\ell_1, \ell_2, \ldots, \ell_n\}$ of links, where each link ℓ_v represents a communication request from a sender s_v to a receiver r_v, what is the largest subset that can transmit simultaneously, given inter-link interference? We study this fundamental problem, known as the capacity problem [1].

In the specific variation of the problem we study, we are also given, for every $\ell_v \in L$, a transmission power $P_v > 0$. The powers received from senders to receivers are defined by an $(n \times n)$-dimensional gain matrix G with positive entries. Specifically, the signal received from s_v at r_w is $G_{wv} \cdot P_v$. Thus an instance in this model can be described by the tuple (L, P, G) where P is the vector of the power assignments P_v for all ℓ_v.

Links that communicate simultaneously necessarily interfere with each other. We use the SINR or *physical* model of interference, known to capture reality with higher fidelity than graph-based models [2–4], which has recently gained substantial attention in the analysis of wireless networks. In this model, a receiver r_v successfully receives a message from a sender s_v if and only if the following condition holds:

$$\frac{G_{vv} \cdot P_v}{\sum_{\ell_w \in S \setminus \{\ell_v\}} G_{vw} \cdot P_w + N} \geqslant \beta, \tag{1}$$

where N is a universal constant denoting the ambient noise, $\beta \geqslant 1$ denotes the minimum SINR (signal-to-interference-noise-ratio) required for a message to be successfully received, and S is the set of concurrently scheduled links in the same *slot*. We say that a link ℓ_v is feasible in S if Eq. (1) is satisfied for ℓ_v. A set S is feasible if each of its links is feasible.

Note that what we described above, without restrictions on the gain matrix G, is known as the *abstract* SINR model. In the more commonly studied *geometric* SINR model, G_{vw} is a polynomial function of $d(s_w, r_v)$, where $d(x, y)$ is the distance between two points x and y. Our results naturally apply to that model as well. The geometric SINR model does not capture

☆ This research was supported in part by the Icelandic Research Fund (grant 90032021).

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0304-3975/$ – see front matter
http://dx.doi.org/10.1016/j.tcs.2013.09.035

obstacles, reflections and other real life distortions, and it has been observed experimentally that signal receptions do not follow simple geometric rules [5,6]. It is thus interesting to see what can be proven in the abstract model.

Our setting where the powers are given as part of the input is often called the *fixed* power case, as opposed to the *power control* case where the algorithm can choose the power assignment. So far, research on fixed power has focused on *oblivious* power assignments, where the power of a link is a (usually simple) function of the length of the link [7–10]. Recently, a constant factor approximation algorithm to find the capacity in the power control case has also been achieved [11]. Unfortunately, none of these techniques appear to extend to the case of arbitrary fixed powers (for either arbitrary or geometric gain matrices). Yet, the problem of arbitrary fixed powers is not only natural, but has practical relevance, as commercial hardware often does not have the capacity of choosing precise powers to implement either an arbitrary assignment *à la* [11], or to implement many of the oblivious power assignments found in literature. Additionally, power may have been pre-selected for other reasons, such as energy availability, in other parts or layers of the system.

In this paper, we prove the following theorem.

Theorem 1. *Let (L, P, G) be an instance of the capacity problem in the abstract SINR model, satisfying $|\mathrm{OPT}| > \frac{1}{2}(1+\epsilon)|L|$ for some $\epsilon > 0$, where* OPT *is a maximum feasible subset of L using P. Then, there is a polynomial time randomized algorithm to find a feasible subset of L of size $\Omega(\epsilon|L|)$, with probability $1 - o(1)$.*

For instances with a smaller optimum, we cannot provide any guarantees.

We do this by means of a semi-definite programming relaxation, which we show how to successfully round if the condition $|\mathrm{OPT}| > \frac{1}{2}(1+\epsilon)|L|$ holds. In addition, we discuss numerical experiments we have performed. These experiments show that the algorithm appears to work quite well on random instances, even better than the guarantees of Theorem 1.

Semi-definite programming has been a staple in designing approximation algorithms for NP-hard problems ever since the seminal work of Goemans and Williams on the Max-CUT problem [12]. It is interesting to note that the discrete "classical" problems closest to wireless capacity, namely the independent set problem and the graph coloring problem, have been fruitfully studied using semi-definite programming [13,14]. The vertex cover problem, also relevant via its connection to the independent set problem, also has SDP-based approximation algorithms [15,16]. Given this background, one may expect some of the techniques to easily carry over to the capacity problem. Yet that does not appear to be the case, at least not in a straightforward manner. A study of the aforementioned papers reveal that the discreteness of the problem plays an important role in the bounds. For example, in [14], the analysis proceeds by bounding the probability of vectors representing edges not being cut by a random hyperplane. Given the additive nature of the SINR model, it is not obvious how to extend that analysis to this case. There have also been a number of results for these problems on hypergraphs [17–19]. Though hypergraphs appear to be closer in spirit to the additive wireless model, they are still different, because the effect of each node on any other node doesn't change in the SINR model (as opposed to in a hypergraph, where it can be different based on which edge they are in). Thus, the (sophisticated) methods on hypergraphs do not appear to translate immediately to the SINR model either. Our SDP relaxation and rounding algorithms are quite simple in contrast to some of the previously mentioned work. Whether or not advanced techniques can be extended to the SINR model remains to be seen.

1.1. Related work

Moscibroda and Wattenhofer [20] were the first to study of the *scheduling complexity* of arbitrary set of wireless links. Early work on approximation algorithms produced approximation factors that grew with structural properties of the network [21–23].

The first constant factor approximation algorithm was obtained for capacity problem for uniform power in [1] (see also [7]) in $\mathbf{R}^2$ with $\alpha > 2$. Kesselheim obtained a $O(1)$-approximation algorithm for the capacity problem with power control for doubling metrics [11]. Around the same time, the first constant factor algorithm for all sub-linear, length monotone power assignments was achieved on general metrics [10]. Other recent studies in the SINR model include work on topological maps [24], distributed algorithms for scheduling [25], distributed power control [26] and auction based spectrum allocation [27].

Other recent works on wireless capacity include [28–30].

We note that the term capacity has a different meaning in the stochastic networking literature (e.g. [31]).

2. SDP-based algorithm

Our algorithm (Algorithm 1) consists of two simple steps. First we solve a semi-definite program, which we show to be a relaxation of the capacity problem. We then randomly select a subset of the links based on SDP solution. The feasible links of this set is then output as the solution.

Some notation is needed. Vectors are denoted by $\vec{x}, \vec{s_w}$, etc. The standard 2-norm of the vector $\vec{x}$ is $\|\vec{x}\|$. The ith entry of $\vec{x}$ is $\vec{x}(i)$. The inner product of vectors $\vec{x}$ and $\vec{y}$ is denoted $(\vec{x} \cdot \vec{y})$. Define $g_{vv} = P_v G_{vv} - \beta N$ and $g_{vw} = P_w G_{vw}$ for $v \neq w$. Note that we can assume without loss of generality that $g_{vv} \geqslant 0, \forall v$. Let OPT be a feasible subset of L of maximum size. Note that $n = |L|$.

Consider the following program.