Theoretical Computer Science 550 (2014) 21–35

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Complexity of finding maximum regular induced subgraphs with prescribed degree

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ARTICLE INFO

Article history:
Received 31 October 2013
Received in revised form 19 March 2014
Accepted 8 July 2014
Available online 17 July 2014
Communicated by P. Widmayer

Keywords:
Bipartite graph
Chordal graph
Graph algorithm
Inapproximability
Planar graph
Regular induced subgraph
Treewidth

ABSTRACT

We study the problem of finding a maximum vertex-subset S of a given graph G such that the subgraph $G[S]$ induced by S is r-regular for a prescribed degree $r \geq 0$. We also consider a variant of the problem which requires $G[S]$ to be r-regular and connected. Both problems are known to be NP-hard even to approximate for a fixed constant r. In this paper, we thus consider the problems whose input graphs are restricted to some special classes of graphs. We first show that the problems are still NP-hard to approximate even if r is a fixed constant and the input graph is either bipartite or planar. On the other hand, both problems are tractable for graphs having tree-like structures, as follows. We give linear-time algorithms to solve the problems for graphs with bounded treewidth; we note that the hidden constant factor of our running time is just a single exponential of the treewidth. Furthermore, both problems are solvable in polynomial time for chordal graphs.

1. Introduction

The problem Maximum Induced Subgraph (MaxIS) for a fixed property Π is the following class of problems [10, GT21]: Given a graph G, find a maximum vertex-subset such that its induced subgraph of G satisfies the property Π. The problem MaxIS is very universal; a lot of graph optimization problems can be formulated as MaxIS by specifying the property Π appropriately. For example, if the property Π is "bipartite," then we wish to find the largest induced bipartite subgraph of a given graph G. Therefore, MaxIS is one of the most important problems in the fields of graph theory and combinatorial optimization, and thus it has been extensively studied over the past few decades. Unfortunately, however, it has been shown that MaxIS is intractable for a large class of interesting properties. For example, Lund and Yannakakis [17] proved that MaxIS for natural properties, such as planar, outerplanar, bipartite, complete bipartite, acyclic, degree-constrained, chordal and interval, are all NP-hard even to approximate.

1.1. Our problems

In this paper, we consider another natural and fundamental property, that is, the *regularity* of graphs. A graph is *r-regular* if the degree of every vertex is exactly $r \geq 0$. We study the following variant of MaxIS:

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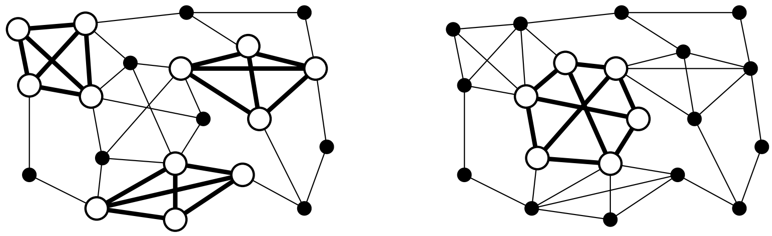
http://dx.doi.org/10.1016/j.tcs.2014.07.008
0304-3975/



(a) (b)

Fig. 1. Optimal solutions for (a) 3-MaxRIS and (b) 3-MaxRICS.

Maximum r-Regular Induced Subgraph (r-MaxRIS)
Input: A graph $G = (V, E)$.
Goal: Find a maximum vertex-subset $S \subseteq V$ such that the subgraph induced by S is r-regular.

The optimal value (i.e., the number of vertices in an optimal solution) to r-MaxRIS for a graph G is denoted by $\mathrm{OPT}_{\mathsf{RIS}}(G)$. Consider, for example, the graph G in Fig. 1(a) as an input of 3-MaxRIS. Then, the three connected components induced by the white vertices have the maximum size of 12, that is, $\mathrm{OPT}_{\mathsf{RIS}}(G) = 12$. Notice that r-MaxRIS for $r = 0$ and $r = 1$ correspond to the well-studied problems maximum independent set [10, GT20] and maximum induced matching [7], respectively.

We also study the following variant which requires the *connectivity* property in addition to the regularity property. (This variant can be seen as the special case of the problem maximum induced connected subgraph for a fixed property Π [10, GT22].)

Maximum r-Regular Induced Connected Subgraph (r-MaxRICS)
Input: A graph $G = (V, E)$.
Goal: Find a maximum vertex-subset $S \subseteq V$ such that the subgraph induced by S is r-regular and connected.

The optimal value to r-MaxRICS for a graph G is denoted by $\mathrm{OPT}_{\mathsf{RICS}}(G)$. For the graph G in Fig. 1(b), which is the same as one in Fig. 1(a), the subgraph induced by the white vertices has the maximum size of six for 3-MaxRICS, that is, $\mathrm{OPT}_{\mathsf{RICS}}(G) = 6$. Notice that r-MaxRICS for $r = 0, 1$ is trivial for any graph; it simply finds one vertex for $r = 0$, and one edge for $r = 1$. On the other hand, 2-MaxRICS is known as the longest induced cycle problem which is NP-hard [10, GT23].

1.2. Known results and related work

Both r-MaxRIS and r-MaxRICS include a variety of well-known problems, and hence they have been widely studied in the literature [2,8,12,14,18,16,19,20]. Below, let n be the number of vertices in a given graph and assume that $\mathrm{P} \neq \mathrm{NP}$.

For r-MaxRIS, as mentioned above, two of the most well-studied and important problems must be maximum independent set (i.e., 0-MaxRIS) and maximum induced matching (i.e., 1-MaxRIS). Unfortunately, however, they are NP-hard even to approximate. Håstad [13] proved that 0-MaxRIS cannot be approximated in polynomial time within a factor of $n^{1/2-\varepsilon}$ for any $\varepsilon > 0$. Orlovich, Finke, Gordon and Zverovich [20] showed the inapproximability of a factor of $n^{1/2-\varepsilon}$ for 1-MaxRIS for any $\varepsilon > 0$. Moreover, for any fixed integer $r \geq 3$, Cardoso, Kamiński and Lozin [8] proved that r-MaxRIS is NP-hard.

For r-MaxRICS, that is, the variant with the connectivity property, Kann [14] proved that longest induced cycle (i.e., 2-MaxRICS) cannot be approximated within a factor of $n^{1-\varepsilon}$ for any $\varepsilon > 0$. Recently, Asahiro, Eto and Miyano [2] gave an inapproximability result for general r: r-MaxRICS cannot be approximated within a factor of $n^{1/6-\varepsilon}$ for any fixed integer $r \geq 3$ and any $\varepsilon > 0$.

A related problem is finding a maximum subgraph which satisfies the regularity property but is not necessarily an induced subgraph of a given graph. This problem has been also studied extensively: for example, it is known to be NP-complete to determine whether there exists a 3-regular subgraph in a given graph [10, GT32]. Furthermore, Stewart proved that it remains NP-complete even if the input graph is either planar [21,22] or bipartite [23].

1.3. Contribution of the paper

In this paper, we study the problems r-MaxRIS and r-MaxRICS from the viewpoint of graph classes: Are they tractable if input graphs have a special structure?

We first show that r-MaxRIS and r-MaxRICS are NP-hard to approximate even if the input graph is either bipartite or planar. Then, we consider the problems restricted to graphs having a "tree-like" structure. More formally, we show that both