Theoretical Computer Science 550 (2014) 36–50

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

On non-progressive spread of influence through social networks

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A R T I C L E I N F O

Article history:
Received 11 December 2012
Received in revised form 15 June 2014
Accepted 8 July 2014
Available online 17 July 2014
Communicated by R. Gavaldà

Keywords:
Spread of influence
Non-progressive
MinPTS problem
NP-hardness
Approximation algorithms

A B S T R A C T

The spread of influence in social networks is studied in two main categories: progressive models and non-progressive models (see, e.g., the seminal work of Kempe et al. [8]). While the progressive models are suitable for modeling the spread of influence in monopolistic settings, non-progressive models are more appropriate for non-monopolistic settings, e.g., modeling diffusion of two competing technologies over a social network. Despite the extensive work on progressive models, non-progressive models have not been considered as much. In this paper, we study the spread of influence in the non-progressive model under the strict majority threshold: given a graph G with a set of initially infected nodes, each of which gets infected at time τ iff a majority of its neighbors are infected at time $\tau - 1$. Our goal in the *MinPTS* problem is to find a minimum-cardinality initial set of infected nodes that would eventually converge to the steady state where all nodes of G are infected.

We prove that while the MinPTS problem is NP-complete for a restricted family of graphs, it admits a constant-factor approximation algorithm for power-law graphs. We do so by proving the lower and upper bounds on the optimal solution of the MinPTS problem in terms of the minimum and maximum degrees of nodes in the graph. The upper bound is achieved in turn by applying a natural greedy algorithm. Our experimental evaluation of the greedy algorithm also shows its superior performance compared to other algorithms for a set of real-world graphs as well as the random power-law graphs. Finally, we study the convergence properties of these algorithms and show that the non-progressive model converges in at most $O(|E(G)|)$ steps.

1. Introduction

Studying the spread of social influence in networks under various propagation models is a central issue in social network analysis [1–4]. This issue plays an important role in several real-world applications including the viral marketing [5–8]. As categorized by Kempe et al. [8], there are two main types of influence propagation models: the progressive and the

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http://dx.doi.org/10.1016/j.tcs.2014.07.009
0304-3975/

non-progressive models. In the progressive models, infected (or influenced) nodes will remain infected forever, but in the non-progressive models, under some conditions, infected nodes may become uninfected again. In the context of the viral marketing and diffusion of technologies over social networks, the progressive model captures the monopolistic settings where one new service is propagated among nodes of the social network. On the other hand, in the non-monopolistic settings, multiple service providers might be competing to get people adopting their services, and thus users may switch among two or more services back and forth. As a result, in these non-monopolistic settings, the more relevant model to capture the spread of influence is the non-progressive model [9–12].

While the progressive model has been studied extensively in the literature [8,13–18], the non-progressive model has not received much attention. In this paper, we study non-progressive influence models, and report both theoretical and experimental results for them. We focus on the strict majority propagation rule in which the state of each node at time τ is determined by the states of the majority of its neighbors at time $\tau - 1$. As an application of this propagation model, consider two competing technologies (e.g., IM service) that are competing in attracting nodes of a social network to adopt their services, and nodes tend to adopt a service that the majority of their neighbors have already adopted. This type of influence propagation process can be captured by applying the strict majority rule. Moreover, as an illustrative example of the linear threshold model [8], the strict majority propagation model is suitable for modeling the transient faults in fault tolerant systems [19–21], and also used in verifying convergence of consensus problems on social networks [22]. Here, we study the non-progressive influence model under the strict majority rule. In particular, we are mainly interested in the minimum perfect target set problem where the goal is to identify a target set of nodes to infect at the beginning of the process so that all nodes get infected at the end of the process. We will present approximation algorithms and prove hardness results for the problem as well as experimental evaluation to validate our results. As our main contributions, we provide the improved upper and lower bounds on the size of the minimum perfect target set, which in turn, result in a constant-factor approximations for power-law graphs. Finally, we also study the convergence rate of our algorithms and report some preliminary results. Before stating our results, we define the problem and the model formally.

Problem formulations. Consider a graph $G(V, E)$. Let $N(v)$ denote the set of neighbors of node v, and $d(v) = |N(v)|$. Also, let $\Delta(G)$ and $\delta(G)$ denote the maximum and minimum degrees of nodes in G respectively.

A *0/1 initial assignment* is a function $f_0 : V(G) \to \{0, 1\}$. For any 0/1 initial assignment f_0, let $f_\tau : V(G) \to \{0, 1\}$ $(\tau \geq 1)$ be the state of nodes at time τ and $t(v)$ be the threshold associated with node v. For the strict majority model, the threshold $t(v) = \lceil \frac{d(v)+1}{2} \rceil$ for each node v.

In the non-progressive strict majority model:

$$f_\tau(v) = \begin{cases} 0 & \text{if } \sum_{u \in N(v)} f_{\tau-1}(u) < t(v) \\ 1 & \text{if } \sum_{u \in N(v)} f_{\tau-1}(u) \geq t(v). \end{cases} \tag{1}$$

In the progressive strict majority model:

$$f_\tau(v) = \begin{cases} 0 & \text{if } f_{\tau-1}(v) = 0 \text{ and } \sum_{u \in N(v)} f_{\tau-1}(u) < t(v) \\ 1 & \text{if } f_{\tau-1}(v) = 1 \text{ or } \sum_{u \in N(v)} f_{\tau-1}(u) \geq t(v). \end{cases} \tag{2}$$

The strict majority model is related to the linear threshold model in which $t(v)$ is chosen at random and is not necessarily equal to $\lceil \frac{d(v)+1}{2} \rceil$.

A 0/1 initial assignment f_0 is called a perfect target set (PTS) if for a finite τ, $f_\tau(v) = 1$ for all $v \in V(G)$, i.e., the dynamics will converge to the steady state of all 1's. The cost of a target set f_0, denoted by $cost(f_0)$, is the number of nodes v with $f_0(v) = 1$. The minimum perfect target set (MinPTS) problem is to find a perfect target set with the minimum cost. The cost of this minimum PTS is denoted by $PPTS(G)$ and $NPPTS(G)$ respectively for the progressive and non-progressive models. This problem is also called target set selection [23]. Another variant of this problem is the maximum active set problem [23] where the goal is to find at most k nodes to activate (or infect) at time zero such that the number of finally infected nodes is maximized.

A graph is power-law if and only if its degree distribution follows a power-law distribution asymptotically. That is, the fraction $P(x)$ of nodes in the network having the degree x goes for large number of nodes as $E[P(x)] = \alpha x^{-\gamma}$ where α is a constant and $\gamma > 1$ is called power-law coefficient. It is widely observed that most social networks are power-law [24].

Our results and techniques. In this paper, we study the spread of influence in the non-progressive model under the strict majority threshold. We present approximation algorithms and hardness results for the problem as well as experimental evaluation of our results. As our main contribution, we provide improved upper and lower bounds on the size of the minimum perfect target set, which in turn, result in improved constant-factor approximations for power-law graphs. In addition, we prove that the MinPTS problem (or computing $NPPTS(G)$) is NP-hard for a restricted family of graphs. In particular, we prove lower and upper bounds on $NPPTS(G)$ in terms of the minimum degree ($\delta(G)$) and the maximum degree ($\Delta(G)$) of nodes in the graph, i.e., we show that

$$\frac{2n}{\Delta(G)+1} \leq NPPTS(G) \leq \frac{n\Delta(G)(\delta(G)+2)}{4\Delta(G) + (\Delta(G)+1)(\delta(G)-2)}.$$