Theoretical Computer Science 568 (2015) 84–96

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

On the self-stabilization of mobile oblivious robots in uniform rings [☆]

Fukuhito Ooshita [a,*], Sébastien Tixeuil [b,c,d]

[a] Graduate School of Information Science and Technology, Osaka University, 1-5 Yamadaoka, Suita, Osaka 565-0871, Japan
[b] Sorbonne Universités, UPMC Univ. Paris 06, UMR 7606, LIP6, F-75005, Paris, France
[c] CNRS, UMR 7606, LIP6, F-75005, Paris, France
[d] Institut Universitaire de France, France

ARTICLE INFO

Article history:
Received 9 December 2013
Received in revised form 6 December 2014
Accepted 15 December 2014
Available online 23 December 2014
Communicated by D. Peleg

Keywords:
Mobile robots
Rings
Gathering
Orientation
Set formation

ABSTRACT

We investigate self-stabilizing algorithms for anonymous and oblivious robots in uniform ring networks, that is, we focus on algorithms that can start from any initial configuration (including those with multiplicity points). First, we show that no probabilistic self-stabilizing gathering algorithm exists in the asynchronous (ASYNC) model or if only global-weak and local-strong multiplicity detection is available. This impossibility result implies that a common assumption about initial configurations (no two robots share a node initially) is a very strong one.

On the positive side, we give a probabilistic self-stabilizing algorithm for the gathering and orientation problems in the semi-synchronous (SSYNC) model with global-strong multiplicity detection. With respect to impossibility results, those are the weakest system hypotheses. In addition, as an application of the previous algorithm, we provide a self-stabilizing algorithm for the set formation problem. Our results imply that any static set formation can be realized in a self-stabilizing manner in this model.

1. Introduction

Background and motivation Studies for mobile robot networks have emerged recently in the field of distributed computing. Their goal is to achieve some tasks by a team of mobile robots with weak capabilities. Most studies assume that robots are identical (robots execute the same algorithm and cannot be distinguished by their appearance) and oblivious (robots have no memory and cannot remember the history of their execution). In addition, it is assumed that robots cannot communicate with other robots directly. The communication among robots is done in an implicit way having each robot observe the positions of others.

Since Suzuki and Yamashita presented a pioneering work [1], many results about such robots have been published. In this paper, we focus on unoriented anonymous ring networks since algorithms for ring networks give solutions for the essential difficulties that arise in robot networks such as symmetry breaking. The main interest of previous works on ring networks is to characterize the minimum assumptions that allow deterministic algorithms. Many algorithms for fundamental problems

☆ A preliminary extended abstract of this paper appears in the proceedings of the 14th International Symposium on Stabilization, Safety, and Security of Distributed Systems (SSS2012).

* Corresponding author. Tel.: +81 6 68794117, fax: +81 6 68794119.
E-mail addresses: f-oosita@ist.osaka-u.ac.jp (F. Ooshita), Sebastien.Tixeuil@lip6.fr (S. Tixeuil).

http://dx.doi.org/10.1016/j.tcs.2014.12.008
0304-3975/

such as the gathering problem, which requires all robots to gather at a non-predetermined single node, have been proposed on various weak assumptions.

Most previous works in the discrete model (*i.e.*, the graph model) for robots make the assumption that initial robot positions are *unique*, that is, in the initial configuration, no two robots share the same node. Still, it is generally accepted that robot algorithms, due to the obliviousness of the robots, are "almost" self-stabilizing, that is, they can recover from an arbitrary initial global state. Characterizing what "almost" means in this context is the topic of this paper, and our goal is to clarify the set of problems that can be solved in a self-stabilizing setting, considering classical hypotheses in robot networks (deterministic *vs.* probabilistic, asynchronous *vs.* semi-synchronous model, global *vs.* local multiplicity detection, strong *vs.* weak multiplicity detection, etc.). Obviously, in an arbitrary initial configuration where occupied nodes host the same number of robots and are symmetric, no deterministic protocol can break the symmetry, which is needed for solving *e.g.* gathering or orienting a ring. Hence, only probabilistic algorithms may be self-stabilizing.

Our contribution In the first part of this paper, we investigate the difficulty of probabilistic self-stabilizing algorithms using weak assumptions. In more details, we show that no probabilistic self-stabilizing algorithm exists that achieves gathering in the asynchronous (ASYNC) model or if only local-strong and global-weak multiplicity detection is available. This impossibility means that the assumption about initial configurations made in previous works is very strong: removing it requires many additional assumptions. Simply put, previous robot algorithms on graphs are *not* "almost" self-stabilizing in the sense that initial configurations are essential to enable the existence of such algorithms under weak assumptions. Interestingly, the impossibility results hold even if we assume rings are oriented (or all robots have the same chirality). That is, the assumption of orientation is not strong enough to allow self-stabilization though the assumption is helpful for many problems.

In the second part, we investigate which problems can be solved in a self-stabilizing manner, and we focus on the gathering and orientation problems. Those two problems are essential to solve most tasks in mobile robot networks. In fact, the difficulty of most problems comes from the lack of agreement on robots' views, *i.e.*, no origin and no orientation in the network. For example, consider the problem that deploys a minimal independent set (MIS) of robots in the network. It is easy for robots to recognize the same form of a MIS from the number of nodes in the ring network. If the network has some origin and orientation, robots can easily recognize the corresponding nodes that construct a MIS. However, if the network has no origin or no orientation, robots cannot uniquely recognize the same nodes due to symmetry of the network configuration even though they know the exact shape of a MIS for a particular number of robots and ring size. Construction of a global origin and orientation is realized by the gathering and orientation problem.

We give probabilistic self-stabilizing algorithms for the gathering and orientation problems on unoriented anonymous rings. We assume the weakest possible model with respect to impossibility results: we assume the semi-synchronous (SSYNC) model and global-strong multiplicity detection. First, we give a self-stabilizing gathering algorithm that achieves gathering in $O(n \log k)$ expected asynchronous rounds and $O(kn)$ expected moves, where k is the number of robots and n is the number of nodes. Since the gathering requires $\Omega(kn)$ moves from initial configurations where robots are evenly scattered, the proposed algorithm is asymptotically optimal for the number of moves. Second, we give a self-stabilizing orientation algorithm using the gathering algorithm. This algorithm not only provides an orientation of the ring but also extracts ℓ robots from the robot pool created by the gathering algorithm, where ℓ is the number of robots required to solve an application problem. This algorithm works if $k \geq \ell + 2$, and requires $O((\log k + \ell)n)$ expected asynchronous rounds and $O(kn)$ expected moves. Finally, as an application of the proposed algorithms, we provide a self-stabilizing algorithm for the set formation problem. The set formation problem can form any static set such as a uniform distribution and a MIS. For the set of size s such that $s \leq k - 1$, this algorithm requires $O((\log k + s)n)$ expected asynchronous rounds and $O(kn)$ expected moves.

Related works Many results about a network of mobile robots have been published since Suzuki and Yamashita presented the pioneering work [1]. They formalize a network of mobile robots in two-dimensional Euclidean space, which is known as the continuous model. They give possibility and impossibility results of the gathering and convergence problem, and characterize the class of geometric patterns that robots can form. Note that they prove that two robots cannot achieve gathering deterministically in the model. Consequently any deterministic gathering algorithm assumes some conditions on the number of robots or the initial positions. Dieudonné and Petit [2] show that, with global-strong multiplicity detection, a deterministic self-stabilizing gathering algorithm exists in the continuous model if and only if the number of robots is odd. Probabilistic self-stabilizing gathering algorithms in the continuous model are proposed in [3,4]. Recently, algorithms for robots with other assumptions such as limited visibility [5], fat robots [6], fat robots with slim cameras [7], and visible lights [8] have been considered in the continuous model.

On the other hand, there are many algorithms for mobile robots in the discrete model (*i.e.*, the graph model). For grid networks, D'Angelo et al. [9] propose deterministic gathering algorithms in the asynchronous model without multiplicity detection. For ring networks, Klasing et al. [10,11] propose deterministic gathering algorithms in the asynchronous model with global-weak multiplicity detection. They also show that there exist some initial configurations where any deterministic algorithm cannot achieve gathering. D'Angelo et al. propose a single algorithm that achieves gathering for any solvable initial configuration in the model [12]. Izumi et al. [13] provide a deterministic gathering algorithm with local-weak multiplicity detection. The algorithm assumes that initial configurations are non-symmetric and non-periodic, and the number of robots is less than half number of nodes. For odd number of robots or odd number of nodes in the same model, Kamei et al. [14,15]