Theoretical Computer Science 546 (2014) 10–16

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Causality in physics and computation

Prakash Panangaden

School of Computer Science, McGill University, Canada

ARTICLE INFO

Keywords:
Causal structure
Event structure
Spacetime
Petri nets

ABSTRACT

Glynn Winskel has had enormous influence on the study of causal structure in computer science. In this brief note, I discuss analogous concepts in relativity where also causality plays a fundamental role. I discuss spacetime structure in a series of layers and emphasize the role of causal structure. I close with some comparisons between causality in relativity and in distributed computing systems.

1. Introduction

Several years ago, in 1982 to be exact, I decided to abandon a career in relativity and quantum mechanics and retrain myself as a theoretical computer scientist. I, like most of my physics colleagues of the time, was completely ignorant about the field. Indeed many physicists had no idea that there was such a field. I recall one of them saying to me, "Theoretical computer science! What is that about? Do you study ideal spherical computers?"

Initially, I was thinking rather unenthusiastically about job security and visa status rather than being excited about a new intellectual adventure. I found two documents that changed that dramatically. The first was an article by Lamport [1] called "Time, clocks and the ordering of events in a distributed system" and the other was Glynn Winskel's remarkable thesis [2]. Both made me realize that I could think about my new subject mathematically and grapple with the foundational questions that I loved in physics.

Since then Winskel and I become friends and have shared many exciting scientific discussions and drinks in pubs. I can think of no better way of celebrating his continued youthful vigour by offering this little note that reflects some of the ways in which he influenced (and influences) me the most. So "Happy birthday Glynn!"

2. The spacetime canvas

This section is necessarily brief; for a detailed treatment of the background mathematics I recommend the excellent book by Hawking and Ellis [3] and the equally excellent but terse monograph by Penrose [4].

The fundamental unit of physics is the *event*. This is taken as a primitive undefined concept but one can think of it as an idealization of a process as the duration and spatial extent of the process shrinks to zero. It is the spatio-temporal analogue of an idealized point. The modern presentation of classical general relativity posits the existence of a smooth 4-manifold of events on which is defined a local "metric" which specifies infinitesimal distances; this is called the *spacetime metric* and the entire structure: manifold together with this metric, is called *spacetime*.

The metric alluded to above is not like a metric that one studies in topology or analysis: it is rather the analogue of a Riemannian metric in geometry. Rather than attributing distances to pairs of points it gives lengths of infinitesimal curves; one can integrate this metric along a curve to obtain a length for a curve.

http://dx.doi.org/10.1016/j.tcs.2014.02.041
0304-3975/2014 Published by Elsevier B.V.

The reason that the word "metric" appears in quotation marks is that unlike the metrics that mathematicians and computer scientists are used to, the spacetime metric takes on positive and negative values and is zero even for many curves connecting pairs of distinct points. The reason for this is the existence of independent events: events that cannot influence each other. Such pairs of events are said to be *spacelike* and the distances are said to be positive. Other pairs of events are possibly causally related and the distances between them are negative: such events are said to be *timelike* related. In order to give a coherent presentation of the structure of spacetime it is best to imagine it as a blank canvas on which more and more sophisticated mathematical structures are defined in successive layers.

As a prelude to painting the spacetime canvas I will quickly review the pre-Einstein–Minkowski picture of spacetime. Here there is a 4-dimensional manifold M of events. A manifold is a topological space so one understands what is meant by open and closed sets. Given two events A and B is it possible for A to influence B? For a fixed A there is a set of events that A can potentially influence: call it $F(A)$, the future of A. There is a set of events that can influence A: call this $P(A)$ the past of A. These two sets are *open* and share a *common* boundary: call this $N(A)$. The set $N(A)$ is "now" as far as A is concerned: it is the set of events that are *simultaneous* with A. The fact that the past and the future share a common boundary means that the points that are pairs of points to the future and past of A that are arbitrarily close to each other and arbitrarily far from A. All this testifies to the lack of any limit on the speed with which causal influences can propagate.

This structure can be neatly described by a real-valued function $t : M \to \mathbb{R}$ called *time*. For all points in $N(A)$ t takes on the same value and for all points to the past of A, t is strictly less than $t(A)$ while for all points in $F(A)$, t is strictly greater than $t(A)$. The manifold has been decomposed into a product of a 3-manifold called S (space) and $\mathbb{R}$ (time): thus $M = S \times \mathbb{R}$. The geometry of spacetime can thus be reduced to the geometry of S which is spatial and one tends to ignore time when talking about geometry. The metric on space is a positive-definite (i.e. Riemannian) metric.

The Einstein–Minkowski picture of spacetime is very different because of the *experimental fact* that the speed of light is constant in all reference frames and the concomitant *belief* that this represents an upper bound on the speed of propagation of *signals*. I now turn to the task of painting the spacetime canvas.

At the most primitive level, spacetime is just a set. At the next level it is a topological space: one has a notion of "nearly" without any metrical connotations and one understands continuity. It is at this level that one encodes the 4-dimensionality and the fact that locally every point looks like $\mathbb{R}^4$. Again the 4 is an experimental fact; perhaps more refined experiments will reveal in the future that it is really 11 dimensional or not even locally homeomorphic to any open subset of any $\mathbb{R}^n$.

The next structure that one imposes is *differential* structure. This allows one to do differential calculus and define smooth curves and tangent vectors to curves. Every point (event) p now has attached to it a 4-dimensional real vector space T_p call the *tangent space* at p. The whole assembly of all these vector spaces held together by being attached to the points of the manifold is called the *tangent bundle*.

The next structure is the crucial one for causality. First a preliminary definition.

Definition 2.1. A subset C of a real vector space V is called a *cone* if

1. $v \in C$ and $-v \in C$ implies $v = 0$,
2. $\forall r \in \mathbb{R}^+, v \in C; r \cdot v \in C$,
3. $\forall u, v \in C; u + v \in C$.

A vector $u \in C$ that can be written as $v + w$ where both v and w are in C and v and w are not scalar multiples of each other is said to be in the *interior* of the cone. A vector not in the interior of the cone is said to be on the *boundary*.

At every point p, there is a pair of subsets C_p^+ and C_p^- of the tangent space called the *future and past light cones*. Each of these sets are *cones* as defined just above. In pictures, one draws the cones as if they were on spacetime itself but they really live in the tangent spaces. That is why it is necessary to define the differential structure first. A vector in the interior of the future (past) light cone at p is said to be a future-pointing (past-pointing) timelike vector. A vector on the boundary of C^+ (C^-) is said to be a future-pointing (past-pointing) *null* vector.

In order for the subsequent discussion to get off the ground one makes a basic assumption about the light cone structure. It is assumed that it is possible to define a notion of future-pointing and past-pointing cones that vary continuously and are defined globally. Such a spacetime is said to be *time-orientable*. One can construct counter-examples to time orientability by using Möbius-strip like constructions; we will assume time orientability as a basic axiom of spacetimes henceforth.

A (smooth/continuous) curve is just a (smooth/continuous) map γ from $\mathbb{R}$ to M or $[0, 1]$ to M if one is considering a curve with end points.

Definition 2.2. A curve is said to be *timelike* if its tangent vector is everywhere timelike. A curve is said to be *causal* if its tangent vector is everywhere timelike or null.

The discussion is best couched in terms of piecewise smooth curves.