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Branching cells for asymmetric event structures

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ABSTRACT

This paper introduces branching cells as elementary units of independent choices in the model of Asymmetric Event Structures (AES), extending a previous work on branching cells for Prime Event Structures. Branching cells consist of subAES of the surrounding AES. Their maximal configurations are shown to tile any maximal configuration of the surrounding AES in a dynamic way.

Branching cells for AES are developed in order to allow the analysis of an optimization procedure in the context of QoS management of web services, presented in a companion paper. Other applications of branching cells include the ability to add a probabilistic layer to AES in a natural fashion where concurrency meets probabilistic independence of choices in distinct and parallel branching cells.

0. Introduction

Asymmetric Event Structures (AES) introduced in the 1990s [1,2] are a model for computational processes involving concurrency, which extends the model of Prime Event Structures (PES) [3]. They depart from PES mainly by the fact that an event has not only a set of mandatory causes, but also some *possible* causes, modeled by a new type of causality called *asymmetric conflict* (see the references in [2] for earlier models with similar aims). The history of an event has a *locality* property in AES. Indeed, the actual history of an event will differ according to the given computation that involves it (configuration, in the event structures language), since the possible causes of the event may or may not be present in the given computation. AES are shown to unfold so-called *contextual nets* or *nets with read arcs* [4,1,2,5–7] in a non-interleaving semantics, just as PES unfold safe Petri nets. Contexual nets differ from usual safe Petri nets in that the firing of a transition depends not only on the presence of tokens in *resource places*, and which are to be consumed, but also on the mere presence of tokens in a set of *contextual places*, and which are not to be consumed by the firing of the transition. It is this very feature that induces the asymmetry of conflict in the unfolding AES.

As for PES, the computational processes associated with an AES are captured by *configurations*. Since configurations are conflict free, it is natural to see a configuration as obtained by different choices, consisting in the resolution of conflicts. The concurrency features of the model, as well as the *confusion* that might appear in it, as described in [3], make it however non-trivial to isolate independent choices. Motivated by probabilistic applications, we have introduced branching cells for PES in this purpose in an earlier work [8]. This paper extends the notion of branching cells to AES, motivated this time by applications in QoS management and orchestration of composite services; this is the topic of the companion paper [9] in this issue.

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The results we obtain are quite similar to the previous ones: we show that branching cells decompose in a dynamic way any maximal configuration of a finite AES, providing a decomposition of the configuration as a partial order of independent choices. These independent choices consist precisely in resolving conflicts *inside* each branching cell in order to select a maximal configuration of the branching cell. Branching cells attached to a given configuration are a tiling, in the sense that they are disjoint. However the entire collection of branching cells of the AES are not disjoint in general, excepted for particular cases such as *trees* (without concurrency) and *confusion free event structures* (restricted concurrency). In contrast, branching cells of a general AES *dynamically tile* configurations. The extension of branching cells from PES to AES is non-trivial mainly for two reasons, that we explain now.

Firstly, the partial order structure on configurations of AES is more subtle than the mere set theoretic subset relation between configurations. This implies than the simple notion of down-closed subset with respect to the causality relation is no longer adequate to provide an “initial view” of the computational process. We therefore introduce choice-complete prefixes (CC-prefix es), which have two keys properties. If C is any configuration, and if U is a CC-prefix, by putting $C_U = C \cap U$ we have that:

1. C_U is indeed an initial sub-configuration of C (which would not hold in general if U was only down closed for the causality, contrasting with PES); and
2. No more choices are pending at the end of the execution of C_U inside U. Informally speaking, the “future” of C_U is entirely disjoint from U.

The above informal mention of the “future” of a configuration is actually made rigorous in the core of the paper, and proves to be an essential tool for the subsequent analysis.

Secondly, in AES, conflict is *asymmetric* and not necessarily binary, whereas in PES, conflict is both symmetric and binary. The fine analysis of choices that we aim at leads us to introduce sources of conflict for AES, generalizing the minimal conflict relation introduced for PES. Sources of conflict are a non-binary relation, but basically play the same role for AES than the minimal conflict relation plays for PES. It is mainly a technicality, and the intuition about minimal conflict relation translates without difficulty into the sources of conflict of AES.

Organization of the paper. Asymmetric Event Structures are presented in Section 1. Our contributions start in Section 2, where we introduce the sources of conflict for AES and the different kinds of prefixes that will be needed in the sequel. Branching cells are introduced in Section 3, first informally described on a simple example. The notion of future of a configuration is introduced afterward; then we state the main result of the paper, which establishes the existence and uniqueness (up to their order) of the decomposition of a maximal configuration through its covering branching cells. Section 4 is devoted to examples illustrating the different notions introduced earlier. In particular, the dynamic character of branching cells is explained on an example. Finally the proof of the main result is the topic of Section 5. After our concluding section (Section 6), two additional sections form an appendix at the end of the paper. In Appendix A, we investigate the applications of branching cells theory to infinite AES, and more specifically to the case of so-called locally finite AES. The topic of Appendix B is to recall through examples the relationship between contextual nets and AES *via* the unfolding theory. It is intended to help the reader in view of the applications presented in companion paper [9].

1. Asymmetric event structures

In this section we follow the presentation of Asymmetric Event Structures (AES) from [2]. In view of our application, we restrict ourselves to *finite* AES; the extension to a class of infinite AES is discussed in Appendix A.

For any set X, we denote by $\mathcal{P}_{\text{fin}}(X)$ the set of finite subsets of X. A *relation* on finite sets of X is a subset $\mathscr{R} \subseteq \mathcal{P}_{\text{fin}}(X)$, with the intuition that a finite subset A of elements of X are $\mathscr{R}$-related if $A \in \mathscr{R}$.

If $(E, \leqslant)$ is a partially ordered set we put $\lfloor x \rfloor = \{y \in E \mid y \leqslant x\}$ for any element $x \in E$, and more generally $\lfloor A \rfloor = \bigcup_{x\in A} \lfloor x \rfloor$ for $A \subseteq E$. We say that a subset $U \subseteq E$ is *$\leqslant$-left closed* if $x \in U \Rightarrow \lfloor x \rfloor \subseteq U$, or equivalently if $\lfloor U \rfloor = U$.

Let $(E, \leqslant, \nearrow)$ be a triple such that $(E, \leqslant)$ is a partially ordered set and $\nearrow$ is a binary relation on E. A relation $\mathscr{R}$ on finite sets of E is said to be a *conflict* relation for $(E, \leqslant, \nearrow)$ if:

1. *($\mathscr{R}$ is $\leqslant$-inherited)*:

$$\forall A \in \mathcal{P}_{\text{fin}}(E)\ \forall x, y \in E \quad \big(A \cup \{x\} \in \mathscr{R}\big) \wedge (x \leqslant y) \quad \Rightarrow \quad A \cup \{y\} \in \mathscr{R}.$$

2. *($\mathscr{R}$ contains the $\nearrow$-cycles)* For any integer $n \geqslant 1$ and for any elements $x_1, \dots, x_n \in E$:

$$x_1 \nearrow x_2 \nearrow \cdots \nearrow x_n \nearrow x_1 \quad \Rightarrow \quad \{x_1, \dots, x_n\} \in \mathscr{R}.$$

If $(\mathscr{R}_i)_{i\in I}$ is any nonempty family of conflict relations, then $\bigcap_{i\in I} \mathscr{R}_i$ is obviously a conflict relation. Since $\mathcal{P}_{\text{fin}}(E)$ is itself a conflict relation, it follows that there exists a smallest conflict relation, that we call *the* conflict relation associated to