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Revisiting the categorical interpretation of dependent type theory

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ABSTRACT

We show that Hofmann's and Curien's interpretations of Martin-Löf's type theory, which were both designed to cure a mismatch between syntax and semantics in Seely's original interpretation in locally cartesian closed categories, are related via a natural isomorphism. As an outcome, we obtain a new proof of the coherence theorem needed to show the soundness after all of Seely's interpretation.

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1. Introduction

About thirty years ago, Seely [23] explained how to interpret extensional Martin-Löf's type theory in locally cartesian closed categories, using the substitution-as-pullback paradigm of categorical logic. But there was a coherence issue arising in this interpretation from the pseudo-functoriality of pullbacks, that had not been addressed by Seely.

In [7], the first author of the present paper studied this problem carefully. He proved the soundness of Seely's interpretation by first designing a syntax with explicit coercions (thus mirroring the pseudo-functoriality at the level of the language being modelled), and then by showing the coherence as a syntactic result, using rewriting techniques. The observation made by Huet in his (unpublished) lecture notes on category theory [14] that Mac Lane's proof of coherence for monoidal categories was a "categorification" of Knuth–Bendix lemma was instrumental for this proof.

In [12], the third author of this work circumvented the coherence issue by showing how to obtain a split model (that is, a model in which composition of substitutions in types and terms is associative "on the nose" rather than up to isomorphism) of Martin-Löf's type theory from a locally cartesian closed category. Then the original type theory can be interpreted straightforwardly in this "strictified" model (see e.g. [13]). The strictification consisted in taking a well-known construction in fibred category theory, going back to Giraud [11] and Bénabou [2], of a right adjoint to the forgetful functor from split fibrations and strict morphisms on a fixed base category to the category of fibrations and fibration morphisms (which are required to preserve the chosen structure only up to isomorphism), and in showing that this construction carries over to deal with additional structure required for the interpretation of Martin-Löf's type theory. Hofmann worked not with fibrations explicitly but rather with the more "syntax-friendly" framework of Cartmell's categories with attributes.

Therefore, in retrospect, the first and the third author had taken "dual" routes to cure the mismatch between the (strict) syntax and the (non-strict) models: either "unstrictify" the syntax, or strictify the model. The genesis of this work lies there:

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we wanted to understand the conceptual architecture in which these two approaches can be linked. In conversations with the second author, it soon became clear that three large categories were involved:

1. a category of non-strict structures and functors preserving the structure up to iso: this is where locally cartesian closed categories and Seely's original interpretation live;
2. a category of strict structures and strict morphisms (i.e., preserving the structure exactly): this is where Hofmann's interpretation lives;
3. a category of non-strict structures and strict morphisms: this is where Curien's modified syntax lives as a free structure.

In pictures, we shall represent the respective morphisms pictorially using

$$\rightsquigarrow \qquad \dashrightarrow \qquad \longrightarrow$$

This three-fold superstructure comes up in various contexts, starting with monoidal categories (and indeed the monoidal case served us as a very useful test bed for the results presented here). In our case, the structures under consideration are the comprehension categories that have products and strong sums, and support extensional identity types, or $\mathbb{ML}$-categories for short. These are fibrations with additional structure, which we shall recall later (Sections 5.1 and 9). But the global picture can emerge without opening this "black box". Let us denote the corresponding three large categories by $\mathbb{ML}$, $\mathbf{S}\mathbb{ML}_\mathbf{s}$ and $\mathbb{ML}_\mathbf{s}$, respectively. Let $\mathtt{Synt}^e$ be the classifying $\mathbb{ML}$-category and let $\mathtt{Synt}$ be the classifying strict $\mathbb{ML}$-category, which are built up from the syntax with explicit coercions and from the original syntax of Martin-Löf's type theory, respectively (see Section 5.3). They are initial in $\mathbb{ML}_\mathbf{s}$ and $\mathbf{S}\mathbb{ML}_\mathbf{s}$, respectively. Our story then goes as follows.

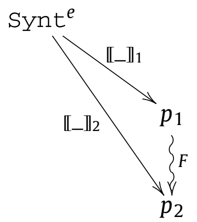
I. Let p_1 and p_2 be $\mathbb{ML}$-categories. Let $[\![\_]\!]_1$ and $[\![\_]\!]_2$ be the interpretation functions of the explicit syntax in p_1, p_2, respectively. Thus we have (cf. item (3) above):

$$[\![\_]\!]_1 \in \mathbb{ML}_\mathbf{s}\big[\mathtt{Synt}^e, p_1\big] \qquad [\![\_]\!]_2 \in \mathbb{ML}_\mathbf{s}\big[\mathtt{Synt}^e, p_2\big]$$

(note that by design interpretation functions are strict). Consider further a morphism

$$F \in \mathbb{ML}[p_1, p_2].$$

Since F is not required to be strict, we do not have $F \circ [\![\_]\!]_1 = [\![\_]\!]_2$, but, as we shall show, the two functors are still related through a natural isomorphism γ. In picture:

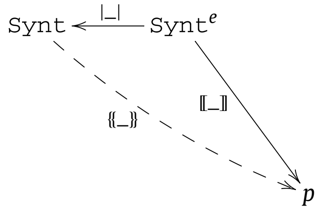


where the triangle commutes up to the isos γ.

II. Let us denote with $\{\!\{\_\}\!\} \in \mathbf{S}\mathbb{ML}_\mathbf{s}[\mathtt{Synt}, p]$ the interpretation function of the original syntax in a strict $\mathbb{ML}$-category p (cf. item (2) above), and let us write more suggestively $|\_|$ for $[\![\_]\!]_{\mathtt{Synt}} \in \mathbb{ML}_\mathbf{s}[\mathtt{Synt}^e, \mathtt{Synt}]$ (indeed, $|\_|$ removes the explicit coercions from the syntax). Then, by initiality in $\mathbb{ML}_\mathbf{s}$ (noting that a fortiori $\{\!\{\_\}\!\} \in \mathbb{ML}_\mathbf{s}[\mathtt{Synt}, p]$), we have the following factorisation:

$$[\![\_]\!] = \big\{\!\!\big\{|\_|\big\}\!\!\big\}$$

or, pictorially:



where the triangle commutes exactly.

III. Let $\mathbb{C}$ be a locally cartesian closed category, which viewed as a fibration $p_1 = \mathtt{cod} : \mathbb{C}^\to \to \mathbb{C}$ endowed with a trivial identity comprehension structure is an object of $\mathbb{ML}$. Let p_2 be the Giraud–Bénabou–Hofmann strictification of p_1. Then there is a faithful and *non-strict* functor $F = (F_t, F_b) : p_1 \to p_2$ over $\mathbb{C}$ (i.e., $p_2 \circ F_t = F_b \circ p_1$ and $F_b = id$).

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