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Touring a sequence of disjoint polygons: Complexity and extension

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In the Touring Polygons Problem (TPP) there is a start point s, a sequence of simple polygons $\mathcal{P} = (P_1, \dots, P_k)$ and a target point t in the plane. The goal is to obtain a path of minimum possible length that starts from s, visits in order each of the polygons in $\mathcal{P}$ and ends at t. This problem was introduced by Dror, Efrat, Lubiw and Mitchell in *STOC '03*. They proposed a polynomial time algorithm for the problem when the polygons in $\mathcal{P}$ are convex and proved its NP-hardness for intersecting and non-convex polygons. They asked as an open problem whether TPP is NP-hard when the polygons are pairwise disjoint. In this paper, we prove that TPP is also NP-hard when the polygons are pairwise disjoint in any L_p norm even if each polygon consists of at most two line segments. This result complements approximation results recently proposed for the touring disjoint polygons problem. As a similar problem, we study the touring objects problem (TOP) and present an efficient polynomial time algorithm for it. This problem is similar to TPP but instead of polygons, we have solid polygonal objects that the tour cannot pass through.

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1. Introduction

A natural and well-studied problem in computational geometry is to find the *shortest path* from a start point s to a target point t, having special properties in the plane. In some applications, the shortest path must visit a set of regions according to a given order. The Zoo-keeper [7], Safari [19], and Watchman route [3,18,20] problems are famous examples of such applications. In the fixed-source version of the Safari and Zoo-keeper problems, we are given a simple polygon P, a start point s inside it and a set of disjoint convex polygons (cages) $\{P_1, \dots, P_k\}$ inside P each of which sharing exactly one edge with P. In the Zoo-keeper problem, we seek a closed tour of minimum possible length that visits the cages only at their boundaries but never enters any of them, while in the Safari problem the tour can enter the cages.

In the Watchman route problem (fixed-source version), we have a simple polygon P and a start point s inside it. The goal is to find a shortest closed tour from s inside P such that every point in P can be seen from at least one point of the tour. It is not difficult to show that in Zoo-keeper and Safari problems, the shortest tour must visit the cages in the same order as they lie on the boundary of P and in Watchman route problem, the shortest tour must visit its *essential pockets* in their order around P [3]. What all these problems have in common is that we should find a shortest path visiting some polygons in a given order. The Touring Polygons Problem (TPP) is the general problem having this visiting property. In TPP, we are given a start point s, a sequence $\mathcal{P} = (P_1, \dots, P_k)$ of simple polygons and a target point t in the plane and the goal is to find the shortest path that starts from s, visits all polygons according to their order and ends at t.

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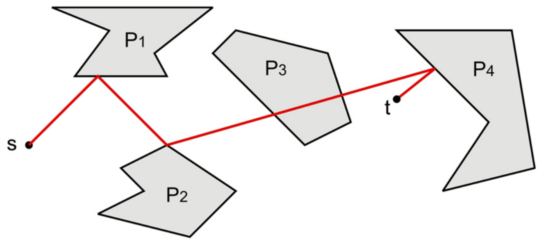

Fig. 1. An instance of TDPP with sequence $(s, P_1, P_2, P_3, P_4, t)$ and its solution.

This problem was introduced by Dror et al. [4] in 2003. They also discussed the constrained version of TPP in which an ordered sequence $\mathcal{F} = (F_0, \ldots, F_k)$ of simple polygons called fences are also given and the portion of the desired path between P_i and P_{i+1} must lie inside F_i (consider s as P_0 and t as P_{k+1}). They proved that TPP is NP-hard in general case and proposed an $O(kn\log(n/k))$ time algorithm for the case where the polygons are convex and pairwise disjoint and an $O(nk^2 \log n)$ time algorithm for the constrained case of convex polygons where n is the total number of vertices of the polygons. But the complexity of TPP (constrained and unconstrained versions) when the polygons are disjoint and allowed to be non-convex has been open even for the L_1 norm. We call this case the Touring pairwise Disjoint Polygons Problem (TDPP). Fig. 1 shows an example of TDPP.

Without determining its complexity, several approximation algorithms have been proposed for this problem [10,11,15]. For example, Pan et al. [15] in 2010 proposed a linear time approximation algorithm for TDPP. Moreover, many similar problems have been recently introduced and investigated. For example in [14], Mitchell and Sharir proved that the 3D Euclidean shortest path problem is NP-complete for the case of obstacles where they are disjoint axis-aligned boxes even if the obstacles are *stacked* axis-aligned quadrants each of which is unbounded to the northeast or southwest. Also, they gave polynomial time algorithm for the case of stacked axis-aligned rectangles that are *terrain-like*, each containing a ray in the (y)-direction, and for other favourable classes of axis-aligned rectangular shapes. Furthermore, they proved that it is NP-hard to decide if the length of an L_1 shortest path from s to t is at most l (where l is a constant) for the following cases: (i) The obstacles are disjoint balls in R^3, or a stack of pancakes (flat circular disks). (ii) The obstacles are a stacked set of squares each at angle $\frac{\pi}{4}$ with respect to the coordinate axes (in this case the problem is NP-complete). As another example, in [16], Polishchuk and Mitchell showed that the optimal touring problem is weakly NP-hard if a lower bound is specified for the length of each segment of the path and noted TDPP as an open problem. In [1], Arkin et al. studied the problem when the polygons are disjoint segments for L_1 norm. In [12], Loffler considered another geometrical problem related to the shortest tour of ordered set of objects. He has proved that given an ordered set of n axis-parallel line segments or squares, it is NP-hard to find a minimum length closed simple tour that visits all segments or squares in order. Another important approach in this subject, is the problem of finding a tour of minimum number of links. In [6], Guibas et al. presented a polynomial time algorithm for obtaining the minimum-link touring path of a sequence of consecutive disjoint convex objects in the plane. More problems and results about finding the shortest path can be found in [5,17].

In Section 2, we prove that TDPP is NP-hard for any L_p norm even if each polygon is composed of at most two joint line segments in which the angles of all segments with the x-axis are in $\{0, \pm\pi/4, \pi/2\}$. Our proof complements the Dror et al. proof of NP-hardness of TPP [4] which is based on the Canny–Reif proof of NP-hardness of the three-dimensional shortest path problem [2].

Similar to the difference between the Zoo-keeper and Safari problems, it is natural to consider TPP when the tour is not allowed to enter the polygons. In this version of TPP, the polygons are considered as solid objects and we call this problem the *touring objects problem* (TOP). In Section 3, we show that this problem can be solved in polynomial time and propose an efficient algorithm to solve it even when the objects are non-convex.

2. Complexity of touring disjoint polygons problem

To prove the NP-hardness of TDPP, we use a reduction from the 3-SAT problem. Let $\Phi(X, C)$ be an instance of the 3-SAT problem where $X = \{x_1, \ldots, x_n\}$ is its variable set and $C = \{C_1, \ldots, C_m\}$ is its clause set such that each variable appears at most once in each clause. We build an instance $\mathcal{P}_\Phi = (s, P_1, \ldots, P_k, t)$ of TDPP with total complexity of $O(n+m)$ and a real number L_Φ in polynomial time such that the solution of TDPP on $\mathcal{P}_\Phi$ is no longer than L_Φ if and only if Φ has a satisfying assignment. For simplicity, we consider the octal Cartesian coordinate system in the plane that is similar to the ordinary Cartesian coordinate system but the x-axis and the y-axis are numbered by octal numbers and the coordinate of each point in the plane is an ordered pair of two octal numbers. We define a *proper path* as a path that starts from s, intersects the polygons of $\mathcal{P}_\Phi$ according to their order and ends at t. An *optimal path* is a proper path of least possible length. $\mathcal{P}_\Phi$ contains the start point s and the target point t as the first and the last polygons, respectively. Between s and t in $\mathcal{P}_\Phi$, there is a sequence (G_i) of *gadgets* each of which consists of a sequence of polygons which are either a line segment or joined two line segments whose angles with the x-axis are in $\{0, \pm\pi/4, \pi/2\}$. So, a proper path should start from s and traverse the gadgets according to their order and end at t. A *semi-optimal path* of G_i is a path that starts from s and optimally traverses G_j gadgets $(1 \le j \le i)$ according to their order. An *incoming semi-optimal path* of G_i is a semi-optimal path of G_{i-1} that

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