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On the equality constraints tolerance of Constrained Optimization Problems

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ABSTRACT

The tolerance value plays an important role when converting equality constraints into inequality constraints in solving Constrained Optimization Problems. Many researchers use a fixed or dynamic setting directly based on trial or experiments without systematic study. As a well-known constraint handling technique, Deb's feasibility-based rule is widely adopted, but it has one drawback as the ranking is not consistent with the actual ranking after introducing the tolerance value. After carefully analyzing how the tolerance value influences the ranking difference, a novel strategy named Ranking Adjustment Strategy (RAS) is proposed, which can be considered as a complement of Deb's feasibility-based rule. The experiment has verified the effectiveness of the proposed strategy. This is the first time to analyze the inner mechanism of the tolerance value for equality constraints systematically, which can give some guide for future research.

1. Introduction

Constrained Optimization Problems (COPs) are very common in real-world applications. The general COPs can be formulated as follows:

$$\begin{aligned} &\text{Minimize } f(\vec{x}) \\ &\text{Subject to: } g_j(\vec{x}) \le 0, \quad j = 1, \cdots, l \\ &\qquad\qquad\quad h_j(\vec{x}) = 0, \quad j = l+1, \cdots, m \end{aligned}$$

where $\vec{x} = (x_1, \cdots, x_n)$ is the decision variable which is bounded by the decision space S. S is defined by the parametric constraints:

$$L_i \le x_i \le U_i, \quad 1 \le i \le n \tag{1}$$

The feasible region Ω is defined by the l inequality constraints $g_j(\vec{x})$ and the $(m-l)$ equality constraints $h_j(\vec{x})$.

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As Evolutionary Algorithms (EAs) are unconstrained search techniques and lack an explicit mechanism to bias the search in the constrained search space, additional mechanisms are needed to deal with constrains when solving COPs [1]. Many constrained optimization evolutionary algorithms (COEAs) are proposed [2–4]. The most popular constraint handling techniques used in COEAs are: methods based on penalty functions, methods based on biasing feasible over infeasible solutions and methods based on multi-objective optimization concepts. As Yao concluded [5], balancing the objective function and constraint violations has always been a key issue in the study of constraint handling.

Regarding the methods based on penalty functions, the infeasible solutions are punished with some extra values adding to the objective function values (i.e., the evaluation function values), and afterwards all the individuals will be ranked according to the evaluation function values. This approach is generic and applicable to any type of constraints. However the subjective setting of various penalty parameters is the main drawback of this method.

To overcome this limitation, some other methods are proposed based on careful comparisons among feasible and infeasible solutions [5,6]. For example, Deb [6] proposed a feasibility-based rule to pair-wise compare individuals:

(1) Any feasible solution is preferred to any infeasible solution.
(2) Among two feasible solutions, the one having better objective function value is preferred.
(3) Among two infeasible solutions, the one having smaller constraint violation is preferred.

Besides, some researchers have employed multi-objective optimization techniques to handle constraints [4,7,8] and got some satisfying results. The multi-objective optimization technique converts the single-objective constraint optimization problem into a bi-objective or multi-objective optimization problem. However, effectively ranking the individuals is still an open problem.

The tolerance value for equality constraints δ plays an important role when converting the equality constraints (i.e., $h_j(\vec{x}) = 0,\ j = l+1, \cdots, m$) into inequality constraints (i.e., $|h_j(\vec{x})| - \delta \le 0,\ j = l+1, \cdots, m$), but the inner working mechanism of δ has not been sufficiently studied. A fixed value (i.e., 0.0001) suggested in [9] has been widely used by some researchers as an evaluation criterion in constraint handling techniques [10–12], while some other researcher simply assume δ as a small constant value [8,12].

The introduction of δ may change the topologic characteristics of the problem as the larger the value is, the more reliable feasible solutions can be found with an easier search process. Therefore, it's very helpful to adopt a large δ at the beginning of the searching. As the evolutionary process continues, a gradually reduced δ will remove the improper solutions and keep the actual optimal value.

Based on this idea, some dynamic setting strategies for the tolerance value δ have been proposed.

This dynamic mechanism is originally proposed in ASCHEA [13], and adopted in [14–17]. Two different settings are available for this mechanism. δ can either be decreased constantly or based on the generation numbers.

For constant decrease, the tolerance value δ is determined by the following expression:

$$\delta_{t+1} = \frac{\delta_t}{\delta'} \tag{2}$$

where δ' is the change rate of δ, often a constant.

In this case, the initial and final values of δ, and the value of δ' are usually decided by the researchers based on experiments.

Mezura et al. [14] suggested δ can be ranged from 0.001 to 0.0004, with 1.00195 of δ'. Especially, for the case of g13, δ was set from 3 to 0.0004, with 1.0145 of δ' to get a feasible solution during the initial generations.

Wang et al. [15] set the initial δ_0 as 3, and the final δ as 5E–06, with δ' equal to 1.0168.

In another paper by Wang et al. [17], they suggested the initial δ_0 should be related with the problem's boundary which can be set as $n \cdot \log 10(\max_{i=1,\cdots,n}(U_i - L_i))$. Here, U_i and L_i are the upper and lower boundary of the ith variable respectively. The final value of δ was set to 0.0001 so as to be in accord with Liang et al. [9].

However, Jia et al. [12] found that both the initial value δ_0 and the change rate δ' proposed by Wang and Cai [17] are problem-dependent, and the setting strategy might not be an effective way to solve a new COP rather than the 24 benchmark functions in Liang et al. [9] based on their experiments. So instead, a constant tolerance value (i.e., 0.0001) was adopted in this paper.

Moreover, Mallipeddi and Suganthan [16] selected the median of equality constraint violations over the entire initial population as the initial δ_0.

Besides the constant decrease of δ mentioned above, some other methods based on generation number have also been presented.

Zavala et al. [18] introduced the percentage of feasible particles (PFP) that is corresponding to the decreasing rate of the tolerance value δ. The value of PFP is updated with the following equation:

$$PFP = \left(1 - \frac{generation}{\max generation}\right)\% \tag{3}$$