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# Histogram-based segmentation of quantum images

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ABSTRACT

In this paper we investigate the use of quantum computing systems in the field of image processing. We consider histogram-based image processing operations and develop quantum algorithms for histogram computation and threshold-based segmentation. The underlying principle used for constructing the proposed quantum algorithms is to reformulate them in order to exploit the performance of the quantum Fourier transform and of quantum amplitude amplification. We show that, compared to the classical correspondents, a significant speedup can be achieved by expressing parts of the computational process in terms of problems that can be solved using these quantum techniques.



## 1. Introduction

Quantum computation and quantum information systems have recently received a lot of attention due to their spectacular perspectives especially regarding performance speedup [1–3] and secure communication [4]. Moreover, the extrapolation of Moore's Law recommends quantum systems as the natural choice for implementing future computing systems as it seems that soon the binary unit of information, the bit, will be implemented at subatomic scale. In the last two decades, scientists have found that the necessary reformulation of information processing in accordance with quantum physics (or Quantum Information Processing) is a tremendously powerful concept for information processing and communication. Efficient quantum algorithms have been formulated allowing for significantly faster calculations than on classical computers. Nevertheless, there are fundamental differences between the algorithms that can be run on quantum computers and those run on classical computers, so a major challenge in quantum computation is to develop efficient quantum algorithms. These differences come from the probabilistic nature of quantum mechanics which stems from the act of measurement: even though a quantum computer can do exponentially many computations in parallel, a measurement of the resulting quantum state yields a random outcome which is not necessarily the particular outcome searched for. The common approach to cope with this behavior is to create constructive interference among the computational paths that lead to 'right' answers and thus high probability of observing those answers. The development of efficient quantum algorithms for practical problems would help in justifying the immense efforts required for building a working quantum computer as it represents a very challenging task that requires spending a huge amount of resources.

The remarkable properties of quantum systems led to the emergence of innovative ideas in all major fields of computing, including graphics processing. A few problems in Computer Graphics have been tackled from the quantum processing perspective and a few quantum algorithms have been suggested for the rendering problem [5,6] and for computational geometry [7]. Even though for rendering it has been shown that quantum solutions can be devised for disparate problems

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such as the visibility of geometric primitives (ex. Z-Buffering), global illumination models (ray tracing, radiosity, photon mapping) or level of detail management, still the full quantum rendering pipeline has not yet been envisioned. Another important result is the quantum RANSAC algorithm [8], a scheme for robust model fitting, which has significant applications in both Computer Graphics and Vision.

As for Quantum Image Processing, the research in the field has encountered fundamental difficulties as it is also still in its infancy. Most of the work done so far refers to fundamental aspects such as representing and storing an image on a quantum computer and the basic processing operations. Representation of color information on one qubit was proposed for the Qubit Lattice approach [9] and was also employed in the FRQI framework [10]. Several basic processing operations were defined in the FRQI framework: geometrical transformations [11], one-qubit quantum gates applied on the color wire [12], a similarity measure between two images based on pixel differences [13]. Two strategies for quantum image watermarking were also developed, one based on restricted geometric transformations [14] and one based on the quantum Fourier transform [15].

Beach et al. [16] show that Grover's quantum search algorithm is applicable to image processing tasks such as pose recognition in a model-based machine vision system.

Other important contributions to the quantum image-processing field rely on the exploitation of maybe the most valuable resource of many-qubit quantum systems, entanglement. It was shown that it could lead to the development of efficient methods for representing and retrieving information about the objects in a quantum image [17] and also to a quantum image compression scheme [18].

Also, it is natural to assume that the image processing field could benefit from the quantum processing transformations (the quantum Fourier transform, the quantum wavelet transform [19] and the quantum discrete cosine transform [20,21]) which seem to be more efficient than their classical counterparts. Using these efficient operations in image processing could underline the potential of the quantum information processing field.

Most of the work done so far in the field of quantum image processing is based on the quantum circuit model of computation. A different approach that uses the adiabatic quantum computing model [22] was proposed in defining solutions for image matching [23] and also for a class of machine learning problems [24], [25].

In this paper we address the problem of representing and segmenting a quantum image. The approach described by Venegas-Andraca and Ball [17] for storing and retrieving information about the shapes of objects in quantum images deals with binary images and thus assumes that the objects be, in fact, already segmented. In contrast, our work provides quantum algorithms for performing the segmentation process on quantum images. In particular we consider the case of threshold-based segmentation. To this end, we first devise a quantum algorithm for computing the image histogram. We show that the representation of quantum images using the method employed by Venegas-Andraca and Bose [9] and Le et al. [10] is not directly suitable for accomplishing this task. Using the quantum histogram we describe the quantum algorithms for image segmentation using two representative techniques for threshold selection: the P-tile method and the iterative thresholding method. All the quantum image processing techniques described in this paper exhibit significant speedups compared to the analogous classical procedures. This is mainly due to the speedup introduced by exploiting the quantum mechanism of amplitude amplification and the quantum Fourier transform.

Before describing the proposed quantum image processing algorithms, background is given to make the paper self-contained. We give a short introduction to the basic concepts in quantum computing and briefly overview the main quantum techniques employed in developing the proposed algorithms: quantum search, quantum phase/eigenvalue estimation, quantum counting. Therefore, the reader familiar with these concepts can skip Sections 2 and 4. In Section 3 we introduce the problem of histogram-based segmentation and overview the most representative classical techniques for threshold selection. Sections 5 and 6 outline the proposed quantum variants for histogram computation and image segmentation, respectively. In Section 7 we summarize our conclusions and discuss the prospects for further development of the quantum image processing field in the light of the new contributions presented in this paper.

## 2. Basic concepts in quantum computing

The quantum analogous of the classical bit is the *qubit*. A qubit is a quantum system whose states can be completely described by the superposition of two orthonormal basis states, labeled $|0\rangle$ and $|1\rangle$ (in a Hilbert space $H = C^2$, $|0\rangle = (1\ 0)^T$, $|1\rangle = (0\ 1)^T$). Any state $|\psi\rangle$ can be described by:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1, \tag{1}$$

where $\alpha$ and $\beta$ represent the probability amplitudes of the basis states, i.e., a measurement of the system yields $|0\rangle$ with probability $|\alpha|^2$ and $|1\rangle$ with probability $|\beta|^2$.

The machine state $|\psi\rangle$ of an $n$-qubit quantum computer is a unit vector in Hilbert space $H = C^{2^n}$:

$$|\psi\rangle = \sum_{i=0}^{2^n-1} \alpha_i |i\rangle, \tag{2}$$

where $\sum_{i=0}^{2^n-1} |\alpha_i|^2 = 1$ and $|\alpha_i|^2$ represents the probability of getting value $|i\rangle$ when measuring the register.

A quantum register $s$ is represented by a sequence of qubits. If $s$ is an $n$-qubit quantum register and $U$ is an operator in the states space $H$, then the operator $U$ applied to the register $s$ is called a *quantum gate*. In closed systems, any quantum