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On the weak computability of a four dimensional orthogonal packing and time scheduling problem

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ABSTRACT

This paper proposes a four dimensional orthogonal packing and time scheduling problem. The problem differs from the classical packing problems in that the position and orientation of each item in the container can be changed over time. In this way, the four dimensional space–time problem better uses the container time. Also, we consider a general case that all parameters are real numbers, which makes the problems more difficult to solve. This paper proposes an algorithm and proves that the algorithm could solve the problem optimally by a finite number of operations. We say this problem is weak computational, meaning that if there exists a universal machine that could represent real numbers and could do unit arithmetic or logical operation on real numbers in finite time, then the algorithm could find optimal solutions in finite time. This paper also presents a proof of the weak computability over a general case of the three dimensional orthogonal packing problem where all parameters are positive real numbers.

1. Introduction

Packing problems [1–3] and scheduling problems [4,5] are important NP-hard problems related to the utilization of space and time, respectively. For some real world applications, such as computer memory allocation, database storage allocation, cookie baking, box refrigeration, etc., can be presented as packing problems. Relevant research includes the four dimensional rectangular packing problem [6], the space scheduling problem [7], the multiprocessor scheduling problem [8], etc.

Items that need to be stored for some time or data that must be preserved for a certain period could change their locations or orientations in the storage area. Such changes make fuller utilization of space and time. Rearrangements are often useful when the storage time is much longer than the rearrangement time. For example, in our daily life, when we take some boxes out of a refrigerator and put some new boxes in, we usually rearrange the boxes that are inside to get a more efficient packing. For some applications the cost associated with the rearrangement is nontrivial, but technological innovations will find ways to overcome this difficulty in the future. By neglecting the rearrangement cost, we abstract a four dimensional (4D) space–time (constructed by three dimensions of space and one dimension of time) packing and scheduling problem, which has potential applications in industry and real life.

As items could change their positions or orientations after being placed into the container and before being taken out, this problem is an extension of the traditional packing problem. We can enhance the flexibility of how to utilize the space and time, and improve the container's space–time utilization by this new model. In [9], we presented an instance of baking biscuits, in which the entire working time of the container was reduced by a third. This new mathematical model is

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expected to achieve a schedule that is not only good but also fast. In this way, relevant applications could be executed more rapidly and economically.

The higher dimensional packing problem is a simplified and preliminary version of this 4D space–time packing and scheduling problem. Valuable works have been done on approximation algorithms and heuristics for two or three dimensional packing problems, including the work of [10–14], etc. For the higher dimensional packing problem, there is also a handful of work published in the literature. Huang and Chen [7] proposed in 1991 that the space scheduling problem could be regarded as a 4D packing problem, and algorithms could be designed basing on an extension of the quasi-physical method for solving two or three dimensional (3D) packing problems [15]. Another early discussion on higher dimensional packing problems was presented in [16], in which Barnes analyzed a case of packing $1 \times 1 \times \cdots \times 1 \times n$ rods. Later on, Fekete et al. discussed a general case of the n-dimensional packing problem [2,17,18] where the parameters are integers. By using a graph-theoretical characterization of feasible packings, they proposed a branch-and-bound framework with new classes of lower bounds, and developed a two-level tree search algorithm for solving the higher dimensional packing problem to optimality. Fekete et al. also reported their computations on two and three dimensional benchmarks. Recently, Harren [19] proposed two approximate schemes for the general d-dimensional hypercube packing problem. Li et al. [6] proposed a greedy heuristic for the 4D rectangular packing problem and generated several 4D packing instances. To the best of our knowledge, there is little work on the 4D space–time packing and scheduling problem.

The two dimensional (2D) rectangular packing problem is strongly NP-hard [20]. Its extension, the three dimensional (3D) cuboid packing problem, has a higher computational complexity. This 4D space–time packing and scheduling problem is based on the 3D cuboid packing problem, adding a continuous parameter of time. Therefore, it is difficult to find out whether this problem is computable or not.

We further consider a general and natural case that all parameters are real numbers. In recent decades, some researchers have studied the computability and computational complexity over real numbers and real functions [21–25]. We prove that if there was a universal machine that could represent real numbers and do unit arithmetic or logical operation in finite time [24], then there exists a deterministic algorithm that could solve the problem by finite operations on these real parameters. We say this problem is weak computational. In this paper, we will present a formal mathematical description for this 4D space–time packing and scheduling problem with real parameters, and then propose a deterministic algorithm that could optimally solve it by finite operations.

2. Problem description and computability analysis

This section presents a formal description of the 4D space–time packing and scheduling problem and the main idea for its weak computability proof.

2.1. Problem description

The 4D space–time packing and scheduling problem can be defined as follows. In a 3D Euclidean space, given a cuboid container with fixed length L, width W and height H, and given n ($n \in N^+$) cuboid items with each item i in size (l_i, w_i, h_i), the time length T_i for each item to be processed is given (all parameters are positive real numbers). The problem is to provide a scheduling plan to determine the following variables:

(1) the starting time S_i of item i (its ending time is $S_i + T_i$);
(2) the position and orientation of item i at every time instant of the interval $[S_i, S_i + T_i)$, which is called the survival period of item i. An item is said to be surviving if it is in the container at the current time.

The goal is to minimize the time that the last item is taken out of the container. We may assume the minimum of S_i to be 0. So the goal is to minimize

$$\max(S_1 + T_1, S_2 + T_2, \ldots, S_n + T_n).$$

At any time instant of the schedule, any surviving item should be located orthogonally and totally within the container with no overlapping of any two surviving items.

2.2. Main idea of the computability proof

If there exists an optimal schedule for the 4D space–time packing and scheduling problem, then naturally it must have an order for the items to be placed in the container. By enumerating all permutations of the items, and by proving that an optimal solution can be found by a greedy strategy for each order, we can prove that the solution with the shortest makespan among all solutions of different permutations is optimal for the original problem.

We prove the weak computability of this problem in three steps. First, we design an exact algorithm A_0 for the cuboid packing decision problem P_0. Then, by using A_0 as a subprocedure, we design an exact algorithm A_1 for problem P_1, the 4D packing and scheduling problem with order constraints. Last, by using A_1 as a subprocedure, we design an exact algorithm A_2 for problem P_2, the original 4D packing and scheduling problem.