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Approximation algorithms on consistent dynamic map labeling ☆

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ABSTRACT

We consider the dynamic map labeling problem: given a set of rectangular labels on the map, the goal is to appropriately select visible ranges for all the labels such that no two consistent labels overlap at every scale and the sum of total visible ranges is maximized. This is also called the *active range optimization* (ARO) problem defined by Been et al. (2006) [2]. We propose approximation algorithms for several variants of this problem. For the *simple ARO problem*, we provide a $3c \log n$-approximation algorithm for unit-width rectangular labels if there is a c-approximation algorithm for the unit-width label placement problem in the plane; and a randomized polynomial-time $O(\log n \log \log n)$-approximation algorithm for arbitrary rectangular labels. For the *general ARO problem*, we prove that it remains NP-complete even for congruent square labels with equal selectable scale range. Moreover, we contribute 12-approximation algorithms for both arbitrary square labels and unit-width rectangular labels, and a 6-approximation algorithm for congruent square labels, and show that the bounds are tight.

1. Introduction

Online maps have been widely used in recent years, especially on portable devices. Such geographical visualization systems provide user-interactive operations such as continuous zooming. Thus, the interfaces provide to a new model in map labeling problems. Been et al. [2,3] initiated an interesting consistent dynamic map labeling problem whose objective is to maximize the sum of total visible ranges, each of which corresponds to the consistent interval of scales at which the label is visible; in other words, the aim is to maximize the number of consistent labels at every scale. In contrast with the static map labeling problem, the dynamic map labeling problem can be considered a traditional map labeling by incorporating *scale* as an additional dimension. During zooming in and out operations on the map, the labeling is regarded as a function of the zoom scale and the map area.

Several desiderata [2] are provided by Been et al. to define this problem. We adopt all desiderata to our problem. Labels are selected to display at each scale and labels should be visible continuously without intersection. Moreover, labels could change their sizes as a function during monotonic zooming at some specific scale. Our goal is to maximize the number of consistent labels at every scale, and thus we maximize the sum of total *active* (visible) ranges to achieve this goal. This

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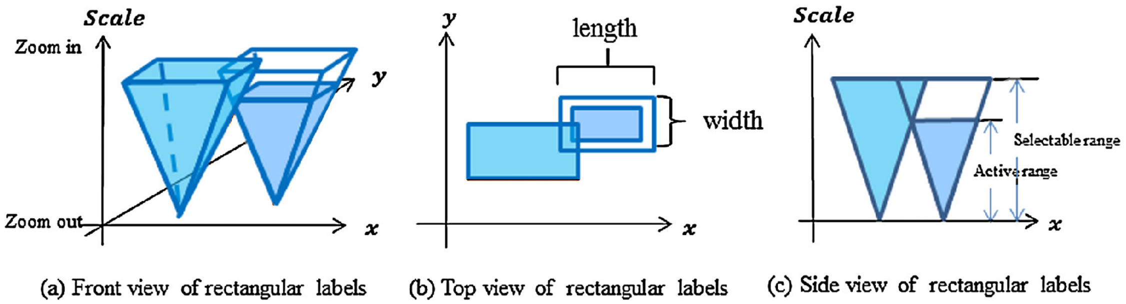

Fig. 1. Two unit-width rectangular labels with selectable ranges and active ranges.

problem is called the active range optimization (ARO) problem. The detailed problem definition is described in the following, and readers may refer to [2,3] for further motivation.

Problem definition. Given a set of n labels to display at every scale, we follow the definition by Been et al.'s work [2,3] and define the *extrusion* E of each label to be the union of all its scaled labels in an open interval $(s_E, S_E) \subseteq (0, S_{\max})$, which we call *selectable range* of E. Note that $S_{\max}$ is an universal maximum scale for all extrusions. Let the set of the n extrusions be $\mathcal{E}$, and the goal is to compute a suitable *active* range $(a_E, A_E) \subseteq (s_E, S_E)$, for each $E \in \mathcal{E}$, where the active range (a_E, A_E) of E, in which its label could be exactly displayed, is a continuous interval lying between its selectable range (see Fig. 1). Actually, when an extrusion E intersects a horizontal plane at scale s, it forms a cross-section; that is, the cross-section is its label L at scale s. Here, we consider invariant point placements with axis-aligned rectangular labels, in which labels always map to the same location, so labels do not slide and rotate.

According to [3], we consider two models in this problem—*general* and *simple*. The general active range optimization (ARO) problem is to choose the active ranges (a_E, A_E) so as to maximize the sum of total active ranges. For the simple ARO problem, it is a variant in which the active range is restricted so that a label is never deselected when zooming in. That is, the active range of a selected extrusion $E \in \mathcal{E}$ is $(0, A_E) \subseteq (0, S_{\max})$.

Moreover, we consider two types of dilation cases in this paper—*proportional dilation* and *constant dilation*. We say that labels have *proportional dilation* if their sizes could change with scale proportionally. In contrast, if the sizes of labels are fixed at every scale, we say that labels have *constant dilation*. For the simple ARO problem with proportional dilation, because we consider rectangular labels, the shapes of extrusions are in fact rectangular pyramids. Let $\pi(s)$ be the hyperplane at scale s. Also let the width and length of the rectangular label $E \cap \pi(s)$ of an (pyramid) extrusion E at scale s be functions $w_E(s) = \frac{s}{S_{\max}} w_E$ and $l_E(s) = \frac{s}{S_{\max}} l_E$, respectively, where w_E and l_E are the width and length of E, respectively, at scale $S_{\max}$. Then, for the general ARO problem with constant dilation, the shapes of extrusions are rectangular prisms. Let width and length be $w_E(s) = w_E$ and $l_E(s) = l_E$, respectively, where $s \in (s_E, S_E)$, because the sizes of all labels are fixed at every scale. In addition, we say that E and $E' \in \mathcal{E}$ intersect at scale s, if and only if $s \subset (s_E, S_E) \cap (s_{E'}, S_{E'})$, $|x_E - x_{E'}| \le \frac{1}{2}(l_E(s) + l_{E'}(s))$ and $|y_E - y_{E'}| \le \frac{1}{2}(w_E(s) + w_{E'}(s))$ are satisfied, where (x_E, y_E) is the central point of a pyramid extrusion E.

Accordingly, our goal is to compute a set of pairwise disjoint truncated extrusions $\mathcal{T} = \{T_E : (a_E, A_E) \mid E \in \mathcal{E}\}$, where T_E is the truncated extrusion of E, so as to maximize the sum of total active range height $\mathcal{H}(\mathcal{T}) = \sum_{E \in \mathcal{E}} |A_E - a_E|$.

Previous work. Map labeling is an important application [9] and a popular research topic during the past three decades [16]. The labeling problems which were proposed before dynamic labeling problems are mostly static labeling problems [3]. There are various settings for static labeling problems [10] and they have been shown to be NP-hard [11]. One of major topics and its typical goal is to select and place labels without intersection and its objective is to maximize the total number of labels. Agarwal et al. [1] presented a PTAS for the unit-width rectangular label placement problem and a $\log n$-approximation algorithm for the arbitrary rectangle case; Berman et al. [5] improved the latter result and obtained a $\lceil \log_k n \rceil$-factor algorithm for any integer constant $k \ge 2$. Then, Chan [7,8] improved the running time of these algorithms. Chalermsook and Chuzhoy [6] showed an $O(\log^{d-2} n \log\log n)$-approximation algorithm for the maximum independent set of rectangles where rectangles are d-dimensional hyper-rectangles.

In addition, there have been a few studies on dynamic labeling. Poon and Shin [15] developed an algorithm for labeling points that precomputes a hierarchical data structure for a number of specific scales. For dynamic map labeling problems, Been et al. [2] proposed several consistency desiderata and presented several algorithms for one-dimensional (1D) and two-dimensional (2D) labeling problems [3]. Note that labels in 1D problems are open intervals; labels in 2D problems are open rectangles. They showed NP-completeness of the general 1D ARO problem with *constant dilation* with square extrusions of distinct sizes, and the simple 2D ARO problem with *proportional dilation* with congruent square extrusions. They focused on dynamic label selection, i.e., assuming a 1-position model for label placement. Moreover, Gemsa et al. [12] provided a FPTAS for general sliding models of the 1D dynamic map labeling problem. Since dynamic map labeling is still a new research topic, there are still many unsolved problems. Yap [17] summarized some open problems.

Our contribution. In this paper, we consider simple 2D ARO with proportional dilation and general 2D ARO with constant dilation. We design a list of approximation algorithms as shown in Table 1. Particularly, we propose the first approximation