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Circular right-angle crossing drawings in linear time ☆

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A R T I C L E I N F O

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A B S T R A C T

A common representational style for drawing graphs is the so-called *circular drawings*, where vertices are represented as points on a circle, and edges are represented as straight line segments. In such drawings, edges may cross; these edge crossings have a negative effect on human readability.

Recent empirical research shows that increasing the angles of edge crossings reduces the negative effect of crossings on human readability. This result has motivated a number of recent investigations of *right angle crossing* graph drawings, where each crossing angle is $\frac{\pi}{2}$.

The main result of this paper is a characterization of graphs that admit a *circular right angle crossing* drawing. We present a linear-time algorithm for testing and constructing such a drawing of a graph, if it exists.

Further, we give an upper bound on the number of edges in a circular right angle crossing drawing, and we note that the optimization problem of constructing circular drawings with large angle crossings can be formulated as a quadratic programming problem.

1. Introduction

Since the late 1970s, researchers have investigated *Graph Drawing*, that is, algorithms to compute "good" drawings of graphs automatically. This study is motivated by the need to visualize real world networks including social networks, computer networks, ER diagrams, PERT networks and biological networks. A collection of optimization criteria and constraints, called "aesthetic criteria", or simply "aesthetics", has been developed. These include minimizing the number edge crossings, maximizing the resolution, minimizing the number of edge bends, and maximizing symmetry; for details, see [2,3].

Perhaps the most significant aesthetic criterion is to minimizing the number of edge crossings in a drawing of a graph. A number of human experiments have established that human understanding is negatively correlated to the number of crossings in a drawing [4]. Consequently, a great deal of research has been directed toward *crossing minimization*, that is, methods to draw graphs with a small number of crossings. See [5] for examples.

However, recent empirical research by Huang et al. [6] shows that the human understanding of a graph drawing increases as the crossing angles increases. Further, if a crossing angle is more than about 70°, then the crossing does not inhibit

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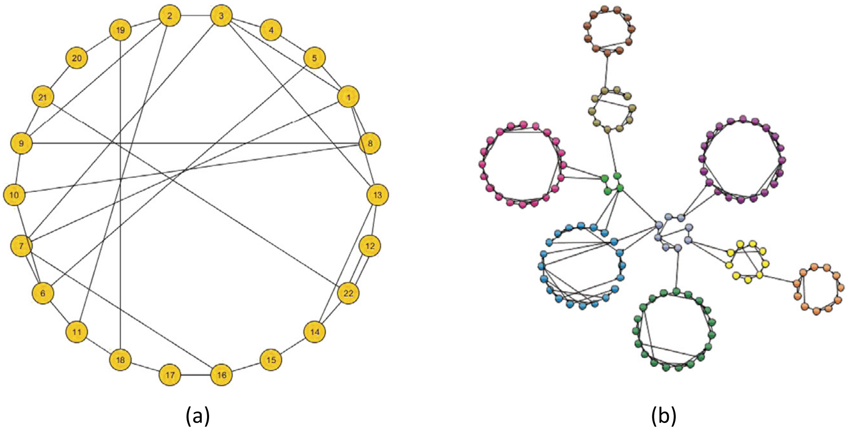

Fig. 1. (a) A circular drawing (from `yWorks` [20]). (b) A graph drawing composed from a number of circular drawings (courtesy of Tom Sawyer Software [19]).

readability at all. This new result has motivated a number of recent investigations into the combinatorics and algorithmics of drawing graphs with large crossing angles. In particular, the concept of *RAC drawing (Right Angle Crossing drawing)* (i.e, each crossing angle is $\frac{\pi}{2}$) has been the subject of intense study.

Didimo et al. [7] introduced the concept of RAC drawing and proved that a straight-line RAC drawing[2] of an n-vertex graph has at most $4n-10$ edges. Argiriou et al. [8] proved that recognizing graphs that have a RAC drawing is $\mathcal{NP}$-hard, and Angelini et al. [9] showed that it is $\mathcal{NP}$-hard to decide whether an acyclic planar digraph admits an upward RAC drawing. Didimo et al. [10] characterized complete bipartite graphs that admit RAC drawings, and Di Giacomo et al. presented a characterization for the two layer RAC drawings [11]. Eades and Liotta [12] showed that a maximally dense RAC drawing (with $4n-10$ edges) is 1-planar, i.e., each edge is crossed at most once. Argyriou et al. studied the problem of *simultaneous RAC drawing* [13].

One of the most popular graph drawing conventions is a *circular drawing*, where each vertex lies on a circle. Crossing minimization in circular layout is well-studied [14–18]. Circular drawings are commonly used in visualization systems for Social Network Analysis. Many commercial graph drawing toolkits (for example, `Tom Sawyer software` [19] and *`yWorks`* [20]) use circular drawing algorithms. Visualizations created from circular drawing algorithms are shown in Fig. 1.

In this paper, we investigate *circular right-angle crossing (C-RAC)* drawings, that is, circular graph drawings in which each crossing is at a right angle. To construct a such a drawing, we need two steps:

1. Crossing reduction: compute an ordering of the vertices around the circle so that the number of crossings is small.
2. Vertex placement: compute the exact locations of the vertices to achieve large crossing angles.

The first step involves an NP-complete problem [21]; however, a number of effective heuristics have been developed. The *CIRCULAR* algorithm of Six and Tollis [15] performs two steps: an initial sequence for the vertices is first created based on the largest outerplanar subgraph, and then the number of crossings is iteratively reduced by careful node moves. Baur and Brandes [18] introduce a simple *sifting* method, also in two phases, that reduces edge crossings by up to 20% in comparison with the CIRCULAR algorithm.

This paper is concerned with the second step. Thus the input of the algorithms described in this paper is the cyclic sequence σ of the vertices around the outer face of the graph.

The main result of this paper is a characterization of *C-RAC graphs*, that is, graphs that admit a C-RAC drawing. We also present a linear-time algorithm for testing whether such a drawing exists for a given cyclic sequence σ of vertices around the outer face of the graph, and constructing such a drawing if it exists.

Further, we give an upper bound on the number of edges in a circular right angle crossing drawing, and we note that the optimization problem of constructing circular drawings with large angle crossings can be formulated as a quadratic programming problem. We describe results of applying the quadratic program approach to a well-known dataset.

The next section defines terminology. Section 3 presents our main result: a characterization of C-RAC graphs together with a linear-time algorithm.

Section 4 gives the upper bound on the number of edges in a circular right angle crossing drawing. Section 5 describes that the problem of constructing circular drawings that maximize crossing angles can be as a quadratic programming problem, and reports results of this approach on a standard dataset.

Section 6 concludes with future research directions.

[2] In this paper graph drawings use straight-line edges unless otherwise stated.