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Timed process calculi with deterministic or stochastic delays: Commuting between durational and durationless actions

Marco Bernardo[a,*], Flavio Corradini[b], Luca Tesei[b]

[a] *Dipartimento di Scienze Pure e Applicate, Università di Urbino, Italy*
[b] *Scuola di Scienze e Tecnologie, Università di Camerino, Italy*

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A B S T R A C T

Several deterministically/stochastically timed process calculi have been proposed in the literature that, apart from their synchronization mechanism, mainly differ for the way in which actions and delays are represented. In particular, a distinction is made between integrated-time calculi, in which actions are durational, and orthogonal-time calculi, in which actions are instantaneous and delays are expressed separately. In a previous work on deterministic time, the two approaches have been shown to be reconcilable through an encoding from the integrated-time calculus CIPA to the orthogonal-time calculus TCCS, which preserves strong timed bisimilarity under certain conditions. In this paper, the picture is completed by first defining a reverse encoding from TCCS to CIPA, which requires slight modifications to both calculi and is shown to preserve only weak timed bisimilarity under conditions tighter than those for the direct encoding. Stochastic time is then addressed, by exhibiting an encoding from the integrated-time calculus MTIPP to the orthogonal-time calculus IML, together with a reverse encoding requiring slight modifications only to the former calculus, with both encodings being shown to preserve strong Markovian bisimilarity under suitable conditions. All the four encodings rely on the assumption that action execution is urgent. Variants are finally discussed in which action execution is delayable, in the sense that an enabled action can let time advance before starting its execution, or only the execution of internal actions is urgent, which is known as maximal progress.

1. Introduction

Computing systems are characterized not only by their functional behavior, but also by their quantitative features. In particular, *timing aspects* play a fundamental role, as they describe the temporal evolution of system activities. This is especially true for *real-time systems*, which are considered correct only if the execution of their activities fulfills certain *temporal constraints*, as well as *shared-resource systems*, in which resource contention determines stochastic fluctuations of the service level measured through suitable *performance indices*. On the modeling side, time is expressed through fixed numbers in the case of real-time systems, yielding a *deterministic* representation of time, or random variables in the case of shared-resource systems, originating a *stochastic* representation of time. In the following, we refer to *abstract time*, in the sense that we use time as a parameter for expressing constraints about instants of occurrences of actions. Unlike physical time, abstract time enables simplifications that allow tractable models to be obtained.

* Corresponding author.
E-mail address: marco.bernardo@uniurb.it (M. Bernardo).

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Many *timed process calculi* have appeared in the literature starting from the late 1980s. Among those with *deterministic delays*, we mention timed CSP [35], temporal CCS [29], timed CCS [39], real-time ACP [2], urgent LOTOS [10], CIPA [1], TPL [20], ATP [32], TIC [34], and PAFAS [14]. As observed in [31,38,12], these calculi differ on the basis of a number of *time-related options*, some of which are recalled below:

- *Durationless actions* versus *durational actions*. In the first case, actions are instantaneous events and time passes in between them; hence, functional behavior and time are *orthogonal*. In the second case, every action takes a certain amount of time to be performed and time passes only due to action execution; hence, functional behavior and time are *integrated*.
- *Relative time* versus *absolute time*. Assume that timestamps are associated with the events observed during system execution. In the first case, each timestamp refers to the time instant of the previous observation. In the second case, all timestamps refer to the starting time of the system execution.
- *Global clock* versus *local clocks*. In the first case, there is a single clock that governs time passing. In the second case, there are several clocks associated with the various system parts, which elapse independent of each other although they contribute to define a unique notion of global time.

Moreover, for these deterministically timed process calculi there are several different *interpretations of action execution*, in terms of whether and when the execution can be delayed, such as:

- *Eagerness*: actions must be performed as soon as they become enabled, i.e., without any delay, thereby implying that their execution is *urgent*.
- *Laziness*: after getting enabled, actions can be delayed arbitrarily long before they are executed.
- *Maximal progress*: enabled actions can be delayed arbitrarily long unless they are independent of the external environment, in which case their execution is urgent.

A comparative study was conducted in [12] by one of the authors, which focuses on two different deterministically timed process calculi obtained by suitably combining the time-related options above. One of the two calculi, TCCS [29], is inspired by the *two-phase functioning principle*, according to which actions are durationless, time is relative, and there is a single global clock. In contrast, the other calculus, CIPA [1], is inspired by the *one-phase functioning principle*, according to which actions are durational, time is absolute, and several local clocks are present. The two considered principles are among the richest combinations of time features [31,38,12], thus allowing for a deep investigation of their relative expressiveness, and the two considered calculi are well-known and paradigmatic instances of the two principles themselves.

In [12], it was shown that the multiple choices concerned with the time-related options are not irreconcilable under the various action execution interpretations. More precisely, a timed-bisimilarity-preserving encoding of CIPA into TCCS was developed for each action execution interpretation. Preservation turned out to depend on certain conditions under action eagerness and to be unconditional under action laziness and maximal progress, thus revealing when the two process calculi have a different expressive power.

In this paper, we complete the previous expressiveness study in two directions. First of all, we consider a reverse encoding from TCCS to CIPA under action eagerness. As pointed out at the end of [12], several issues need to be addressed before such a reverse encoding can be established. Our contribution consists of providing an answer to each of the various issues, which will lead to slight modifications of both calculi. The reverse encoding is proved to be fully abstract with respect to *weak* timed bisimilarity, as opposed to the direct one demonstrated to be fully abstract with respect to *strong* timed bisimilarity in [12].

The second direction that we explore is concerned with process calculi extended with *stochastic delays*, such as for instance MTIPP [18,22], PEPA [25], MPA [11], $\mathrm{EMPA_{gr}}$ [8,5], $\mathrm{S}\pi$ [33], IML [21], and PIOA [36]. In these calculi, delays are no longer expressed through nonnegative numbers, but through nonnegative random variables. The latter typically follow *exponential distributions*, each characterized by a positive real number called *rate*, so that the underlying stochastic processes turn out to be *continuous-time Markov chains*. These models are mathematically tractable [37] and fit well with the interleaving view of concurrency thanks to the *memoryless property* of the considered distributions.

The time-related options and action execution interpretations discussed for deterministically timed calculi apply to a large extent also to such stochastically timed calculi. This is especially true for the difference between durationless and durational actions. In contrast, the distinction between relative and absolute time and the concept of clock are not so important in a Markovian framework due to the memoryless property of exponential distributions, which establishes that the residual time to the termination of an event follows the same exponential distribution as the overall duration of the event.

However, there is a fundamental difference between deterministically timed calculi and stochastically timed calculi in terms of choices among alternative behaviors. In the former calculi, all choices are *nondeterministic* precisely as in classical process calculi, because time passing cannot affect choices according to *time determinism*. In the latter calculi, choices can instead be *probabilistic* when they are based on delays or durational actions, because in that case the *race policy* is adopted, which means that the event sampling the least duration is the one that takes place. For example, in an orthogonal-time setting, the deterministically timed process $(t) \,.\, P_1 + (t) \,.\, P_2$ – where $+$ denotes the alternative composition operator – can