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Conditional fault-tolerant edge-bipancyclicity of hypercubes with faulty vertices and edges

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ABSTRACT

Let F be a faulty set in an n-dimensional hypercube Q_n such that in $Q_n - F$ each vertex is incident to at least two edges, and let f_v, f_e be the numbers of faulty vertices and faulty edges in F, respectively. In this paper, we consider the fault-tolerant edge-bipancyclicity of hypercubes. It is shown that each edge in $Q_n - F$ for $n \geq 3$ lies on a fault-free cycle of any even length from 6 to $2^n - 2f_v$ if $f_v + f_e \leq 2n - 5$. This gives an answer for a problem proposed by Yang et al. (2016) [33].

1. Introduction

A multicomputer system is comprised of a plurality of processors that communicate by exchanging messages via an interconnection network (network for short). It is well known that a topological structure of a network can be modeled by a loopless undirected graph G, where the vertex set $V(G)$ represents the processors and the edge set $E(G)$ represents the communication links. In this paper, we use graphs and networks interchangeably.

An n-dimensional hypercube Q_n (an n-cube for short) is one of the most versatile and efficient architectures yet discovered for building massively parallel or distributed systems. It possesses quite a few excellent properties such as recursive structure, regularity, symmetry, small diameter, relatively short mean internode distance, low degree, and much small link complexity, which are very important for designing massively parallel or distributed systems [2,7,11,18,30,33].

Cycle networks are fundamental networks for parallel and distributed computation which are suitable for developing simple algorithms with low communication cost. Many efficient algorithms were originally designed based on cycles for solving a variety of algebraic problems, graph problems and some parallel applications, such as those in image and signal processing [1,2,7,22,30,32]. Thus, it is important to have an effective cycle embedding in a network. The cycle embedding problem can be briefly stated as follows: How to find a cycle of given length into a given graph. If a graph G contains a cycle of every length from 3 to $|V(G)|$, then it is *pancyclic*, and it is *bipancyclic* if it contains a cycle of every even length from 4 to $|V(G)|$, where $|V(G)|$ is the number of vertices of G. The pancyclicity is an important measurement to determine whether a network is suitable for an application mapping rings of any length into the network [11,16,32]. In a heterogeneous computing system, each edge and each vertex may be assigned with distinct computing power and distinct

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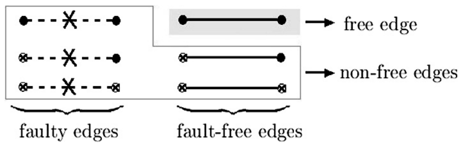

Fig. 1. An illustration of faulty edge (dotted line), fault-free edge, free edge and non-free edge.

bandwidth, respectively [26]. Thus, it is worthwhile to extend pancyclicity to edge-pancyclicity and vertex-pancyclicity [32, 33]. If every edge (or vertex, resp.) of G lies on a cycle of every length form 3 to $|V(G)|$, then G is said to be *edge-pancyclic* (or *vertex-pancyclic*, resp.) and it is *edge-bipancyclic* (or *vertex-bipancyclic*, resp.) if every edge (or vertex, resp.) lies on a cycle of every even length form 4 to $|V(G)|$. A *bipartite graph* is one whose vertex set can be partitioned into two nonempty subsets X and Y such that every edge has one end-vertex in X and the other in Y. Bipancyclicity (or edge-bipancyclicity, vertex-bipancyclicity, resp.) is essentially a restriction of the concept of pancyclicity (or edge-pancyclicity, vertex-pancyclicity, resp.) to bipartite graphs whose cycles are necessarily of even length.

Element (edge or vertex) failure is inevitable when an interconnection network is put in use. Therefore, the fault-tolerant capacity of a network is a critical issue. The problem of fault-tolerant cycle embedding of some networks has received much attention recently [2–17,19–25,27–29,31–34]. Let $F = F_v \cup F_e$ be a faulty set of a graph G, where $F_v \subseteq V(G)$ and $F_e \subseteq E(G)$. Write $f_v = |F_v|$ and $f_e = |F_e|$. A vertex or an edge of G is *faulty* if it is contained in F, and *fault-free* if it is not in F. Note that a faulty edge may have fault-free end-vertices and a fault-free edge may have faulty end-vertices [2]. An edge (x, y) is said to be *free* if it is fault-free and both its end-vertices are fault-free (see Fig. 1). A path (or a cycle) is *fault-free* if it contains neither a faulty vertex nor a faulty edge. We use $G - F$ to denote the subgraph of G which is induced by $\{x \mid x \in V(G) \text{ and } x \notin F\}$ and does not contain the edges in F. Hence, an edge in $G - F$ is indeed a free edge of G. A graph G is *fault-tolerant bipancyclic* (or *fault-tolerant edge-bipancyclic*, *fault-tolerant vertex-bipancyclic*, resp.) if $G - F$ is bipancyclic (or edge-bipancyclic, vertex-bipancyclic, resp.).

For a faulty hypercube Q_n, it may have only faulty vertices, only faulty edges or both faulty vertices and faulty edges. Under the *conditional-fault model*, that is, each fault-free vertex is incident to at least two free edges, Tsai [22,23] proved that each edge in $Q_n - F$ with $n \geq 4$ lies on a fault-free cycle of any even length from 6 to $2^n - 2f_v$ if $f_v \leq n-1$ and $f_e = 0$. For an n-cube Q_n with only faulty edges, Xu et al. [31] proved that each free edge in Q_n with $n \geq 4$ lies on a fault-free cycle of any even length from 6 to 2^n if $f_e \leq n-1$, and latter, Shih et al. [19] or Tsai and Lai [25] proved that Xu et al.'s result also holds even if $f_e \leq 2n-5$ independently. These works suggest us to consider the edge-bipancyclicity of hypercubes with both faulty edges and faulty vertices. Inspired from Shih et al.'s results in [19] and [25], we have the following problem.

Problem 1. Under the conditional-fault model, does each free edge in a faulty hypercube Q_n with $n \geq 4$ lie in a fault-free cycle of any even length from 6 to $2^n - 2f_v$ if $f_v + f_e \leq 2n - 5$?

Most recently, it was shown in [33] that, under the conditional-fault model, each free edge in Q_n with $n \geq 3$ lies on a fault-free cycle of any even length from 6 to $2^n - 2f_v$ if $f_v + f_e \leq 2n - 5$ and $f_e \leq n - 2$. In this paper, we consider the case $f_v + f_e \leq 2n - 5$ and $f_e \geq n - 1$ and an affirmative answer is given for the Problem 1.

The rest of this paper is organized as follows. In the next section, some necessary definitions and notations are introduced, and some initial results are proposed. Section 3 discusses the partition and cycles constructions, which are applied in the proof of main result. The main result is proved in Section 4 and conclusions are given in Section 5.

2. Preliminaries

Let G be a graph. A *walk* in G is a sequence of vertices $(x_0, x_1, x_2, \ldots, x_k)$ such that $(x_i, x_{i+1}) \in E(G)$ for $0 \leq i \leq k-1$. If $x_i \neq x_j$ for every $i \neq j \in \{0, 1, \ldots, k\}$, then the walk is a *path* of length k from x_0 to x_k, and if further $x_0 = x_k$ exceptionally, then the walk is called a *cycle* of length k. We use $P[x, y]$ to denote a path from x to y, called an (x, y)-*path*. Two paths are *vertex-disjoint* if they have no common vertices. A path (or a cycle, resp.) of length k is called a k-*path* (or a k-*cycle*, resp.). A k-cycle is even or odd depending on the parity of k. By P and C, we commonly denote a path and a cycle respectively, and we denote the length of P and C by $|P|$ and $|C|$, respectively. The *distance* between x and y in G is denoted by $d_G(x, y)$, which is the length of a shortest path between x and y in G.

An n-*dimensional hypercube* Q_n is an undirected graph with 2^n vertices each labeled with an n-bit binary string $x_n x_{n-1} \cdots x_2 x_1$, where $x_i = 0$ or 1 for each $1 \leq i \leq n$, and with two binary strings adjacent if they have exactly one-bit distinct. Clearly, Q_n is a bipartite graph. Let $x = x_n x_{n-1} \cdots x_2 x_1$. For $1 \leq k \leq n$, we use x^k to denote the binary string $x_n \cdots x_{k+1}(1 - x_k)x_{k-1} \cdots x_1$. The *Hamming distance* $h(x, y)$ between two n-bit strings x and y is the number of $x_i \neq y_i$, for $1 \leq i \leq n$. It is easy to see that $d_{Q_n}(x, y) = h(x, y)$. An edge (x, y) in Q_n is of *dimension* i if $y = x^i$. The following proposition is straightforward.

Proposition 2.1. *(See [22, Lemma 2].) Let e be an edge in the healthy n-cube Q_n with $n \geq 2$. Then there are $n - 1$ cycles with length 4 containing e in common.*