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Note

Edge-pancyclicity and edge-bipancyclicity of faulty folded hypercubes

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ABSTRACT

Let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the folded hypercube FQ_n so that $|F_v| + |F_e| \leq n-2$, for $n \geq 2$. Choose any fault-free edge e. If $n \geq 3$ then there is a fault-free cycle of length l in FQ_n containing e, for every even l ranging from 4 to $2^n - 2|F_v|$; if $n \geq 2$ is even then there is a fault-free cycle of length l in FQ_n containing e, for every odd l ranging from $n+1$ to $2^n - 2|F_v| - 1$.

1. Introduction

Choosing an appropriate *interconnection network* (*network* for short) is an important integral part of designing parallel processing and distributed systems. There are a large number of network topologies that have been proposed. Among the proposed network topologies, the *hypercube* [1] is a well-known network model which has several excellent properties, such as recursive structure, regularity, symmetry, small diameter, short mean internode distance, low degree, and small edge complexity. Numerous variants of the hypercube have been proposed in the literature [3,4,17]. One variant that has been the focus of a great deal of research is the *folded hypercube*, which can be constructed from a hypercube by adding an edge joining every pair of vertices that are the farthest apart, i.e., two vertices with complementary addresses. The folded hypercube has been shown to be able to improve a system's performance over a regular hypercube in many measurements, such as diameter, fault diameter, connectivity, and so on [3,20].

Since vertices and/or edges in a network may fail accidentally, it is necessary to consider the fault-tolerance of a network. Hence, the issue of fault-tolerant cycle embedding in an n-dimensional folded hypercube FQ_n has been studied in [2,5,7–14, 20,19]. Embedding cycles in networks is important as many network algorithms utilize cycles as data structure. In this paper, let F_v and F_e be the sets of faulty vertices and faulty edges, respectively, in FQ_n. Choose any fault-free edge e. We prove that if $n \geq 3$, there is a fault-free cycle of length l in FQ_n containing e, for every even l ranging from 4 to $2^n - 2|F_v|$; if $n \geq 2$ is even then there is a fault-free cycle of length l in FQ_n containing e, for every odd l ranging from $n+1$ to $2^n - 2|F_v| - 1$.

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Throughout this paper, a number of terms—network and graph, node and vertex, edge and link—are used interchangeably. The remainder of this paper is organized as follows. In Section 2, we provide some necessary definitions and notations, and we present our main result in Section 3. Some concluding remarks are given in Section 4.

2. Basic definitions

A *path* in a graph $G=(V,E)$ is a sequence of distinct vertices so that any two consecutive vertices are joined by an edge, and the *length* of a path is the number of edges in the path. A *cycle* is a path of length at least 3 so that there is an edge joining the first and last vertices of the path, and the *length* of a cycle is the number of vertices in the cycle. For any graph $G=(V,E)$ and vertices $u,v\in V$, we denote the length of a shortest path in G from u to v by $d_G(u,v)$ (if there exists no path from u to v in G then $d_G(u,v)$ is defined as ∞). If C is a cycle of length c in the graph G containing the edge (x,y) and P is a path of length p in G from x to y that contains no vertices of C apart from x and y then we say that the cycle of length $c-1+p$ obtained by removing the edge (x,y) and including the path P is obtained by *grafting* the path P onto the cycle C. Let X be a set of vertices and edges of G. We denote the subgraph of G induced by the vertices of X and the vertices incident with the edges of X by $\langle X\rangle$. All other standard graph-theoretic terminology can be obtained from [21].

If a graph $G=(V,E)$ contains cycles of every length from 3 to $|V|$, then it is *pancyclic*, and it is *bipancyclic* if it contains a cycle of every even length from 4 to $|V|$, where $|V|$ denotes the number of vertices in G.[1] The pancyclicity is an important measurement of whether a network is suitable for an application inquiring cycles of any length within the network [6]. In a heterogeneous computing system, each edge and each vertex may be assigned with distinct computing power and distinct bandwidth, respectively [18]. Thus, it is worthwhile to extend pancyclicity to edge-pancyclicity and vertex-pancyclicity. If every edge (or vertex) of G lies on a cycle of every length from 3 to $|V|$ then G is said to be *edge-pancyclic* (or *vertex-pancyclic*), and G is *edge-bipancyclic* (or *vertex-bipancyclic*) if every edge (or vertex) lies on a cycle of every even length from 4 to $|V|$.

We study graphs $G=(V,E)$ which model interconnection networks in which there might be faulty nodes or faulty links. Such faults are modeled by *faulty vertices* in V, the set of which we denote by F_v, and *faulty edges* in E, the set of which we denote by F_e. Every vertex of $V\setminus F_v$ is called *fault-free* and every edge of $E\setminus F_e$ that is not incident with any vertex of F_v is called *fault-free* (so, any fault-free edge is, by definition, incident only with fault-free vertices). If H is a subgraph of G then $(F_v\cup F_e)\cap H$ denotes the set of vertices of F_v and edges of F_e that lie in H. We say that a cycle or a path in G is *fault-free* if every vertex and edge that lies on the cycle or path is fault-free.

The *hypercube* Q_n has $\{0,1\}^n$ as its vertex set and there is an edge joining two vertices if the vertex names differ in exactly one bit. The *folded hypercube of dimension* n, FQ_n, also has $\{0,1\}^n$ as its vertex set. In FQ_n, there is an edge joining two vertices if the vertex names differ in exactly one bit or in every bit. If an edge is such that the two incident vertices differ in only the ith bit, for some $i\in\{1,2,\ldots,n\}$, then we say that this edge lies in *dimension* i, with the neighbor of a vertex x where the edge lies in dimension i denoted $x^{(i)}$ (this applies to both Q_n and FQ_n); and if an edge is such that the two incident vertices differ in every bit then the edge is called a *complementary edge*, with the neighbor of a vertex x where the edge is a complementary edge denoted $\overline{x}$ (this applies only in FQ_n). Note that it makes sense to write, for example, $x^{(i,j)}$, to denote the vertex obtained by flipping the ith and jth bits of the name of x, and to write, for example, $\overline{x^{(i)}}$ to denote the vertex obtained by flipping every bit of the name of x except the ith. Consequently, the folded hypercube FQ_n is simply the hypercube Q_n with the addition of the complementary edges.

For FQ_n, we can choose some $i\in\{1,2,\ldots,n\}$ and *partition* the folded hypercube over dimension i by separating the vertices whose ith component of their names is 0 from those whose ith component is 1. This results in two hypercubes of dimension $n-1$, denoted $Q_{n-1}^{0,i}$ and $Q_{n-1}^{1,i}$, induced by the vertices whose ith bits are 0 and 1, respectively. We suppress the superscript i if the partition dimension is understood. Of course, the complementary edges of FQ_n form a perfect matching, each incident with exactly one vertex in each hypercube, as do the edges of FQ_n lying in dimension i.

The folded hypercube FQ_n is clearly regular of degree $n+1$ and is known to be $(n+1)$-connected, vertex-transitive and edge-transitive [16,19]. Both Q_n and FQ_n have been extensively studied. In particular, we shall use the following results.

Lemma 1. *(See [15].) Let $n\geq 3$. Let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the hypercube Q_n so that $|F_v|+|F_e|\leq n-2$. Let u and v be any two distinct fault-free vertices in Q_n. There is a fault-free path of length l in Q_n joining u and v, for every l ranging from $d_{Q_n}(u,v)+2$ to $2^n-2|F_v|-1$ where $l-d_{Q_n}(u,v)$ is even.*

Lemma 2. *(See [6].) Let $n\geq 3$. Let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the hypercube Q_n so that $|F_v|+|F_e|\leq n-2$. Choose any fault-free edge e. There is a fault-free cycle of length l in Q_n containing e, for every even l ranging from 4 to $2^n-2|F_v|$.*

Lemma 3. *(See [19].) Let $n\geq 2$. Let F_e be a set of faulty edges in the folded hypercube FQ_n so that $|F_e|\leq n-1$. Choose any fault-free edge e. If $n\geq 3$ then there is a fault-free cycle of length l in FQ_n containing e, for every even l ranging from 4 to 2^n. If $n\geq 2$ is even then there is a fault-free cycle of length l in FQ_n containing e, for every odd l ranging from $n+1$ to 2^n-1.*

[1] The size of any set X of vertices and edges in a graph is denoted $|X|$.