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Changing of the guards: Strip cover with duty cycling ☆

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ABSTRACT

The notion of *duty cycling* is common in problems which seek to maximize the lifetime of a wireless sensor network. In the duty cycling model, sensors are grouped into *shifts* that take turns covering the region in question, and each sensor can belong to at most one shift. We consider the imposition of the duty cycling model upon the Strip Cover problem, where we are given n sensors on a one-dimensional region, and each shift can contain at most $k \le n$ sensors. We call the problem of finding the optimal set of shifts so as to maximize the length of time that the entire region can be covered by a wireless sensor network, k-Duty Cycle Strip Cover (k-DutySC). In this paper, we present a polynomial-time algorithm for 2-DutySC. Furthermore, we show that this algorithm is a $\frac{35}{24}$-approximation algorithm for k-DutySC. We also give two lower bounds on the performance of our algorithm: $\frac{15}{11}$, for $k \ge 4$, and $\frac{6}{5}$, for $k = 3$, and provide experimental evidence suggesting that these lower bounds are tight. Finally, we propose a fault tolerance model and find thresholds on the sensor failure rate over which our algorithm has the highest expected performance.

1. Introduction

We consider the following problem: Suppose we have a one-dimensional region (or interval) that we wish to cover with a wireless sensor network. We are given the locations of n sensors located on that interval, and each sensor is equipped with an identical battery of finite charge. We have the ability to set the sensing radius of each sensor, but its battery charge drains in inverse proportion to the radius that we set. Our goal is to organize the sensors into disjoint coverage groups (or *shifts*), that will take turns covering the entire region for as long as possible. We call this length of time the *lifetime* of the network.

More specifically, we consider the Strip Cover problem with identical batteries under a *duty cycling* restriction. An instance consists of a set $X \subseteq [0, 1]$ of n sensor locations, and a rational number B representing the initial battery charge of each sensor. Each battery discharges in inverse linear proportion to its radius, so that a sensor i whose radius is set to r_i survives for B/r_i time. In the *duty cycling* model, the sensors are partitioned into disjoint coverage groups, called *shifts*, which take turns covering the entire interval for as long as their batteries allow. The sum of these lengths of time is called the *lifetime* of the network and is denoted by T. For any fixed $k \le n$, the k-Duty Cycle Strip Cover (k-DutySC) problem seeks

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an optimal partitioning of the sensors such that the network lifetime T is maximized, yet no coverage group contains more than k sensors. In the fault tolerant variant, each sensor may fail to activate with some fixed probability $p \in [0, 1]$, and we seek to maximize the *expected* lifetime of the network (i.e., the expected sum of lifetimes of surviving shifts).

Motivation Applications of scheduling problems similar to Strip Cover are increasingly common. One such application involves monitoring a fence, or supply line, that exists in inhospitable territory. In this case, it may be feasible (even cost-effective) to deploy a set of sensors along the fence, but unfeasible to position them at pre-determined locations. For example, it might be easy to drop the senors from an airplane, but impossible to dispatch human beings to place them. While the scheduler may have access to the location of each sensor via GPS, technical limitations may require that a single assignment be given. In such a scenario, we might be incentivized to organize the sensors into disjoint shifts, providing motivation for our duty cycling model. Finally, any physical device will have some nonzero failure rate, and thus a fault-tolerant solution will be more robust.

Solutions to the general Strip Cover problem contain both the radial assignments and activation and de-activation times for each sensors. As a result, these solutions can be complicated to implement and understand. Moreover, interdependence among multiple sensors can make such solutions susceptible to catastrophic decline in network lifetime if there is a non-zero probability of sensor failure. Conversely, since in the duty cycling model each sensor can participate in at most one cover shift, the scheduling of the shifts is of little importance. Furthermore, by minimizing the number of sensors participating in each shift, duty cycling solutions become more resilient to sensor failure.

Related work This line of research began with Buchsbaum, et al.'s [5] study of the Restricted Strip Cover (RSC) problem. In RSC, the locations and sensing radii of n sensors placed on an interval are given, and the problem is to compute an optimal set of activation times, so as to maximize the network lifetime. They showed that RSC is NP-hard, and presented an $O(\log \log n)$-approximation algorithm. Gibson and Varadarajan [11] later improved on this result by discovering a constant factor approximation algorithm.

The problem of finding the optimal set of radial assignments for sensors deployed on an interval, rather than the activation times, is more tractable. Peleg and Lev-Tov [12] considered the problem of covering a finite set of m target points while minimizing the sum of the radii assigned, and found an optimal polynomial-time solution via dynamic programming. The situation wherein the whole interval must be covered corresponds to a "one shift" version of n-DutySC, wherein the restriction is not upon the size of each shift, but upon the number of shifts. Bar-Noy, et al. [4] provided an optimal polynomial-time algorithm for this problem.

The interest in duty cycling developed in part from the introduction of the Set k-Cover problem by Slijepcevic and Potkonjak [16]. This problem, which they showed to be NP-hard, seeks to find at least k disjoint covers among a set of subsets of a base set. Perillo and Heinzelman [15] considered a variation in which each sensor has multiple modes. They translated the problem into a generalized maximum flow graph problem, and employed linear programming to find an optimal solution. Abrams et al. [1] provided approximation algorithms for a modification of the problem in which the objective was to maximize the total area covered by the sensors. Cardei et al. [6–8] considered adjustable range sensors, but also sought to maximize the number of non-disjoint set covers over a set of target coverage points.

The work of Pach and Tóth [13,14] also has applications in this context. They showed that a k-fold cover of translates of a centrally-symmetric open convex polygon can be decomposed into $\Omega(\sqrt{k})$ covers. Aloupis, et al. [2] improved this to the optimal $\Omega(k)$ covers, and the centrally-symmetric restriction was later lifted by Gibson and Varadarajan [11]. In each of the above cases, the concept of finding many disjoint set covers, which can be seen as shifts, is used as a proxy for maximizing network lifetime.

Finally, the general Strip Cover problem, in which each sensor has a *different* battery charge, was studied by Bar-Noy, et al. [4]. They also considered the Set Once Strip Cover (OnceSC) problem, in which the radius and activation time of each sensor can be set only once. They showed that OnceSC is NP-hard, and that RoundRobin (sensors take turns covering the entire interval) is a $\frac{3}{2}$-approximation algorithm for both OnceSC and Strip Cover. Bar-Noy, et al. [4] also showed that the approximation ratio of any duty cycling algorithm is at least $\frac{3}{2}$ for both OnceSC and Strip Cover. Bar-Noy and Baumer [3] also analyzed non-duty cycling algorithms for Strip Cover with identical batteries. The Connected Range Assignment problem studied by Chambers, et al. [9], wherein the goal is to connect a series of points in the plane using circles, is also related. They presented approximation bounds for the case where solutions use a fixed number of circles, which is similar to limiting shift sizes.

Our results In Section 2, we define the class of k-DutySC problems, and present the trivial solution to 1-DutySC. We present a polynomial-time algorithm, which we call Match, for 2-DutySC in Section 3. This algorithm is based on the reduction to Maximum Weight Matching in bipartite graphs. In Section 4, we compare the performance of RoundRobin to an algorithm that uses only a single shift. We prove that when the sensors are equi-spaced on the coverage interval, RoundRobin performs most poorly in comparison to the one shift algorithm. Then we study the performance of RoundRobin on these "perfect" deployments. This study is used to analyze Match in k-DutySC, but is of independent interest, since perfect deployments are the most natural. In Section 5 we show that Match is a $\frac{35}{24}$-approximation algorithm for k-DutySC. We also give two lower bounds on the performance of Match: $\frac{15}{11}$, for $k \geq 4$, and $\frac{6}{5}$, for $k = 3$, and provide experimental evidence suggesting that these lower bounds are tight. The question of whether k-DutySC is NP-hard, for $k \geq 3$, remains