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Gathering of robots on anonymous grids and trees without multiplicity detection ☆,☆☆

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ABSTRACT

The paper studies the gathering problem on grid and tree networks. A team of robots placed at different nodes of the input graph, has to meet at some node and remain there. Robots operate in Look–Compute–Move cycles; in one cycle, a robot perceives the current configuration in terms of occupied nodes (Look), decides whether to move toward one of its neighbors (Compute), and in the positive case makes the computed move instantaneously (Move). Cycles are performed asynchronously for each robot. The problem has been deeply studied for the case of ring networks. However, the known techniques used on rings cannot be directly extended to grids and trees. Moreover, on rings, another assumption concerning the so-called *multiplicity detection* capability was required in order to accomplish the gathering task. That is, a robot is able to detect during its Look operation whether a node is empty, or occupied by one robot, or occupied by an undefined number of robots greater than one.

In this paper, we provide a full characterization about gatherable configurations for grids and trees. In particular, we show that on these topologies, the multiplicity detection is not required. Very interestingly, sometimes the problem appears trivial, as it is for the case of grids with both odd sides, while sometimes the involved techniques require new insights with respect to the well-studied ring case. Moreover, our results reveal the importance of structures like grids and trees that allow to overcome the multiplicity detection with respect to the ring case.

1. Introduction

In the field of robot based computing systems, one of the most popular problems is certainly the *gathering*. A pool of robots, initially situated at various locations, has to gather at the same place (not determined in advance) and remain there.

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Many variants of the problem have attracted the interest of numerous researchers (see e.g., [17,22] and references therein). In this paper, we consider the case of anonymous grid and tree networks where anonymous, asynchronous and oblivious robots can move according to the so-called Look–Compute–Move cycles [28]. In each cycle, a robot takes a snapshot of the current global configuration (Look), then, based on the perceived configuration, decides either to stay idle or to move to one of its adjacent nodes (Compute), and in the latter case it makes an instantaneous move to this neighbor (Move). Cycles are performed asynchronously for each robot. This means that the time between Look, Compute, and Move operations is finite but unbounded, and is decided by the adversary for each robot. Hence, robots may move based on significantly outdated perceptions. Robots are oblivious, i.e., they do not have any memory of past observations. Thus, the target node (which is either the current position of the robot or one of its neighbors) is decided by the robot during a Compute operation solely on the basis of the location of other robots perceived during the Look operation. Robots are anonymous and execute the same deterministic algorithm. They cannot leave any marks at visited nodes, nor send any messages to other robots. We remark that the Look operation provides the robots with the entire grid/tree configuration concerning occupied nodes. That is, a robot perceives whether a node of the input graph is occupied or not, but it cannot distinguish how many robots reside on an occupied node.

1.1. Related work

The problem of making mobile entities meet on graphs [4,18,23,28] or open spaces [9,17,30,32] has been extensively studied in the last decades. When only two robots are involved, the problem is usually referred to as the *rendezvous* [2,8,10,18,33]. Under the Look–Compute–Move model, many problems have been addressed, like the *graph exploration* and the *perpetual graph exploration* [3,5,6,19–21], while the rendezvous problem has been proven to be unfeasible on rings [28].

Concerning gathering under the Look–Compute–Move model, much work has been done in the last years for the ring topology. It has been proven that the gathering is unsolvable if the robots are not empowered by the so-called *multiplicity detection* capability [28], either in its *global* or *local* version. In the former type, a robot is able to perceive whether any node of the network is occupied by a single robot or more than one (i.e., a *multiplicity* occurs). In the latter type, a robot is able to perceive the multiplicity only if it is part of it.

Using global multiplicity detection, different types of configurations have required different approaches. In particular, periodicity and symmetry arguments have been exploited. In a ring, a configuration is called *periodic* if it is invariant under non-trivial (i.e., non-complete) rotations. A configuration is called *symmetric* if the ring has a geometrical *axis of symmetry* that reflects single robots into single robots, multiplicities into multiplicities, and empty nodes into empty nodes. In [28], it is proven that, even with the global multiplicity detection, the gathering is unsolvable for two robots, for periodic configurations and for those symmetric configurations where the axis of symmetry passes through two edges. Then, several algorithms have been proposed for different kinds of initial configurations, in detail: for the case of odd number of robots and that of asymmetric configurations [28], for symmetric configurations with an even number of robots greater than 18 [27], and for 4 and 6 robots [12,14,29]. These papers left open some cases which have been addressed in [13] where a unified strategy for all the gatherable configurations has been provided.

Regarding the local weak assumption, in [24], it has been proposed an algorithm for aperiodic and asymmetric configurations with the number of robots k strictly smaller than $\lfloor \frac{n}{2} \rfloor$, with n being the number of nodes composing the ring. In [25], the case where k is odd and strictly smaller than $n-3$ has been solved. In [26], an algorithm for the case where n is odd, k is even, and $10 \le k \le n-5$ is provided. The case of aperiodic and asymmetric configurations has been closed in [15]. Finally, a full characterization of the ring case with local weak multiplicity detection has been provided in [16].

1.2. Our results

In this paper, we fully characterize the gathering on grids and trees. We show that the multiplicity detection capability is not needed.

On grids, in particular, we show that even if the global multiplicity detection is assumed, a configuration is ungatherable only if it is periodic (i.e., the same view can be obtained by rotating the grid around its geometric center of an angle smaller than 360 degrees) on a grid with at least an even side, or it is symmetric with the axis of symmetry passing through edges. For all the other cases, we provide a gathering algorithm which does not require any multiplicity detection except for configurations on 2×2 grids with three robots where the local multiplicity detection is helpful.

On trees, we fully characterize the cases where the configurations are ungatherable even if the global multiplicity detection is assumed. Then, we provide a strategy that solves all the other cases without assuming any multiplicity detection capabilities.

The general strategy followed by our gathering algorithms consists of uniquely identifying a gathering node exploiting the robots' placements, and then move the robots in order to maintain such point. Moreover, the algorithm does not exploit any multiplicity detection because, whenever gathering is possible, we are able to identify such a unique node and to move the robots toward it in such a way that it is maintained.

To our knowledge, grid and tree topologies are the least structured classes of graphs permitting to avoid the multiplicity detection assumption. Moreover, it is worth mentioning that in our solutions many robots can move concurrently, instead of just one or two as it was for the ring case.