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Strong connectivity of sensor networks with double antennae ☆

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ABSTRACT

Inspired by the well-known Dipole and Yagi antennae we introduce and study a new theoretical model of directional antennae that we call *double antennae*. Given a set P of n sensors in the plane equipped with double antennae (with either dipole-like or Yagi-like propagation patterns) of angle ϕ, we study the *connectivity* and *stretch factor* problems, namely finding the minimum range such that there exists an orientation of the double antennae of that range that guarantees strong connectivity or stretch factor of the resulting network. We introduce the new concepts of $(2, \phi)$-connectivity and ϕ-angular range and use them to characterize the optimality of our algorithms. We prove that the ϕ-angular range is a lower bound on the range required for strong connectivity and show how to compute it in time polynomial in n. We give an algorithm for orienting the antennae so as to attain strong connectivity using optimal range when $\phi \geq 3\pi/4$ and an algorithm that approximates the range to $\sqrt{3}$ times the optimal range for $\phi \geq \pi/2$. For $\phi < \pi/3$, we show that the problem is NP-complete to approximate within a factor $\sqrt{3}$. For $\phi \geq \pi/2$, we give an algorithm to orient the antennae so that the resulting connectivity network has a stretch factor of at most 4 compared to the underlying unit disk graph.

1. Introduction

Directional antennae are versatile transceivers which are widely used in wireless communication. With proper design they are known to improve overall energy consumption [14], enhance network capacity [11,21], improve topology control [10], and offer the potential for mitigating various security threats [12], just to mention a few applications. The

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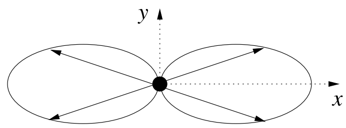

Fig. 1. Radiation pattern of a dipole antenna in the xy-plane.

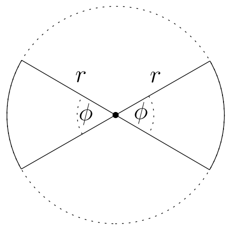

Fig. 2. Double antenna with beamwidth ϕ and range r.

motivation for our present study comes from the work in [5] which introduced the network connectivity problem for directional sensors and provided several algorithms for analyzing angle-range tradeoffs.

Dipole antennae (or *dipoles, for short*) are well-known basic antennae that are commonly used in radio communication. At their simplest, they consist of two straight collinear conductors of equal length separated by a small gap. Moreover, the radiation pattern for such antennae—indicating the strength of the signal in a given direction—in the xy-plane is usually depicted by two equal size closed curves known as *lobes*. Fig. 1 illustrates the variability of the strength of the signal depending on the direction of the beam (see [20]). When the two lobes are not identical with the apex of one of the two lobes being closer to the origin than the other, the resulting antenna radiation pattern corresponds to a *Yagi antenna*, thus indicating that the antenna's transmission range is longer in one direction versus its opposite.

Motivated by the above, we introduce the following theoretical model of *Dipole-like* and *Yagi-like* antennae which we refer to as *double* antennae. These two concepts are captured in the following two geometric definitions.

Definition 1. A (ϕ, r)*-double antenna*, depicted in Fig. 2, is an antenna consisting of two opposite sectors (or beams) each of angle (or beamwidth) ϕ which can send and receive up to a distance r, called the radius (or range) of the antenna.

Definition 2. More generally, a (ϕ, r_1, r_2)*-double* antenna is a double antenna such that the range of one beam is equal to r_1 and the range of the opposite beam r_2.

Clearly, a (ϕ, r_1, r_2)-double antenna is also a $(\phi, \min\{r_1, r_2\})$-double antenna. Unless otherwise specified, in this paper, a double antenna refers to a (ϕ, r)-double antenna.

In this paper we assume that each sensor is equipped with two antennae: a double antenna that is used to transmit and an omnidirectional antenna that is used to receive. Thus, a directed link from a sensor u to a sensor v is formed if v is in the beam of u. This defines the transmission graph. In practice, a double antenna that steers the main lobe towards a specific direction can be implemented, for example, as a dipole antennae with mechanical steering [1], electrical steering [2] or as a circular array antenna [18] by choosing a predetermined beam.

The two problems considered in this paper are related to orienting antennae and can be described in detail as follows.

Problem 1 *(Connectivity problem with double antennae).* Given a set P of sensors in the plane each equipped with one double antenna with beamwidth $\phi \le \pi$, determine the minimum antenna range, denoted by $\hat{r}_\phi(P)$, so that there exists an orientation of the antennae that induces a strongly connected transmission graph.

When the set P of points is clear from the context we simply use the abbreviated notation $\hat{r}_\phi$.

It is worth noting that for a sufficiently small angle ϕ, the problem is equivalent to the well-known bottleneck traveling salesman problem (BTSP) which requires to find a Hamiltonian cycle that minimizes the longest edge. Furthermore, a trivial upper-bound on the antenna range for $\phi \le \pi$ of three times the optimal range can be computed by finding a Hamiltonian cycle with edge length bounded by three times the longest edge of the MST [16, Problem C35.2-4] since the longest edge of the MST is also a lower bound for the orientation problem for any angle ϕ. However, for the BTSP a better analysis given in [19] shows that a 2-approximation can be obtained in polynomial time. Essentially, they proved that the lower bound for the BTSP is at least the longest edge of the 2-connected graph G that minimizes the longest edge. Thus, the 2-approximation is obtained easily since the square of any 2-connected graph is Hamiltonian [9]. In this paper we introduce the concept of $(2, \phi)$-connectivity that generalizes the concept of 2-connectivity in geometric graphs and allows us to give an optimal range algorithm when $\phi \ge 3\pi/4$.