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A logical characterization of extensive games with short sight

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A B S T R A C T

The notion of *short sight*, introduced by Grossi and Turrini, weakens the unrealistic assumption in traditional extensive games that every player is able to perceive the entire game structure. In this paper, we propose a more general model for extensive games with short sight. For reasoning about extensive games with short sight, we propose a new logic language and then present an axiomatization for this logic. We prove the soundness and completeness of the axiomatization. In addition, we show that the logic can formally characterize the solution concepts and Pearce's lemma in games with short sight.

1. Introduction

Orthodox game theory relies on (common knowledge of) perfect decision rationality, i.e. unlimited cognitive and information processing capabilities of players. Even for finite games of perfect information like chess it is, however, obvious that these requirements are far beyond what human decision makers can accomplish. In a chess game, the actual game space is exponential in the size of the game configuration, and may have a computation path too long to be effectively handled. So we often seek sub-optimal solutions by considering only limited information or bounded steps foreseeable by a player that has relatively small amount of computation resources. To develop a more realistic model for extensive games, Grossi and Turrini proposed the concept of *games with short sight* [1], in which players can only see part of the game tree in the sense that they may have no ability to see all the terminal nodes. In other words, players may not be able to calculate the consequences of their actions up to the terminal nodes. Due to this kind of 'short sight', decisions need to be taken even when terminal nodes are not accessible.

The concept of *short sight* is of great importance: First, it relaxes the assumption that the complete game structure is common knowledge in traditional extensive games, and offers a more realistic model for describing the real game. To see this, please consider a game, called Tic–Tac–Toe.[1] Since the number of leaves in the complete game tree is 362,880 (i.e., 9!), only a part of it is shown in Fig. 1. Given such a big tree, it is evident that at each node, say v_0, a player can only predict a part of the future states, rather than the complete game. That is exactly the idea of *short sight.* In games with short sight, a

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[1] The rule for Tic–Tac–Toe is: Two players take turns to mark the spaces in a 3×3 grid. The player who succeeds in placing three respective marks in a horizontal, vertical, or diagonal row wins the game.

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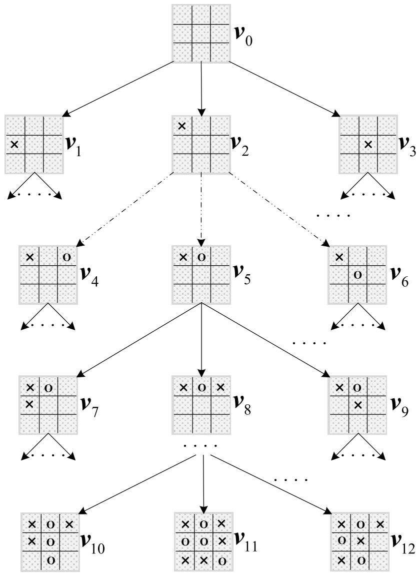

Fig. 1. A Tic–Tac–Toe game.

sight function is introduced to demonstrate what players can foresee (i.e., the part of the nodes they can compute) at each node.

Second, it stimulates the requirement of finding appropriate new solution concepts for extensive games. In standard games, equilibrium concepts can be viewed as a *universal plan*, describing what players will do in every possible situation. This makes sense only because players are presumed to know the complete game tree, and thus to know in advance all the situations that can arise. However, in the presence of *short sight*, this intuition no longer makes sense. Players can only make a *local plan* in the game they can see.

Third, it enables us to design automated players, for whom the complexity of searching a game tree is lower since players only have to compute the situations they can see. In games with short sight, the set of states that players can see at every node is only a small part of the whole future. As a result, it is unnecessary for players to make computations within a 'huge' tree. Thus the actual time cost for searching games with short sight is lower than that for searching traditional extensive games.

Finally, it is applicable to modeling search algorithms in AI, e.g., the $\alpha - \beta$ pruning algorithm. It shows the way in which computer games (and possibly humans as well) explore, evaluate and decide among available moves when the game tree is too large to be fully explored.

A number of interesting issues arise given a new model for extensive games, among which the most fundamental one is representing and reasoning about extensive games with short sight. Many researchers have developed logical systems for traditional extensive games. [2] concentrated on describing equilibrium concepts and strategic reasoning. [3] proposed an epistemic logic to deal with epistemic aspects of extensive games. [4] introduced a logic reasoning about how information or assumptions on the preferences of other players can be used by agents in order to realize their own preferences. The work of [5] on propositional game logic initiated the study of game structure using algebraic properties. [6] used dynamic logic to describe games as well as strategies. [7] studied a logic in which not only are games structured, but also are strategies. [8] worked on the relationship of branching time logic to extensive form games.

However, no works have considered the logical representation for extensive games with short sight. There are good reasons to provide such logic-based representations: First, logical representations can be used together with tools and techniques developed in computer science. For example, logic can express properties φ of extensive games with short sight. Then by the technique of model checking, it can be checked whether a game has property φ. Second, logical representations are usually succinct, and thus offer convenience for automated reasoning such as theorem proving. Third, such a logical system allows characterizing the properties of extensive games formally, e.g., new solution concepts. Finally, it is interesting to see how 'sight' would interact with traditional game structures by investigating what will happen to game logics when 'sight' modality is explicitly introduced.

In this paper, we focus on the logical analysis of game-theoretical notions of the solutions concepts in games with short sight. However, before a logical characterization of games with short sight, one thing that should be established is the introduction of games with short sight. Although the model for games with short sight has been studied in the literature, there is still substantial room for improving it. We are mainly concerned the following two improvements.

• Definition for sight, concerning the variety of short sight among individuals. In [1], all players share one short sight function, which is a function representing the nodes players can see. However, in real game scenarios, the computational