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All-to-all broadcast problems on Cartesian product graphs

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ABSTRACT

All-to-all communication occurs in many important applications in parallel processing. In this paper, we study the all-to-all broadcast number (the shortest time needed to complete the all-to-all broadcast) of Cartesian product of graphs under the assumption that: each vertex can use all of its links at the same time, and each communication link is half duplex and can carry only one message at a unit of time. We give upper and lower bounds for the all-to-all broadcast number of Cartesian product of graphs and give formulas for the all-to-all broadcast numbers of some classes of graphs, such as the Cartesian product of two cycles, the Cartesian product of a cycle with a complete graph of odd order, the Cartesian product of two complete graphs of odd order, and the hypercube Q_{2n} under this model.

1. Introduction

Broadcasting is an information dissemination problem in a connected network in which a message, originated by one vertex, is transmitted to all vertices of the network by placing a series of calls over the communication lines of the network. This is to be completed as quickly as possible, subject to the constraints that each call involves only two vertices, a vertex which already knows the message can only inform one of the vertices to which it is connected, and each call requires one unit of time. This problem has been extensively investigated in recent years for many different networks and under a large variety of models. For an account of the history of the area of broadcasting and the intensive research devoted to it, see the surveys [15] and [7]. For different models of the broadcasting problem, such as the *k-broadcasting* problem, the *line broadcasting* problem, the *multiple originator broadcasting* problem, the *multiple message broadcasting* problem, the *all-to-all personalized communication* problem, see [1,3–6,8–14,16–19] and the references therein.

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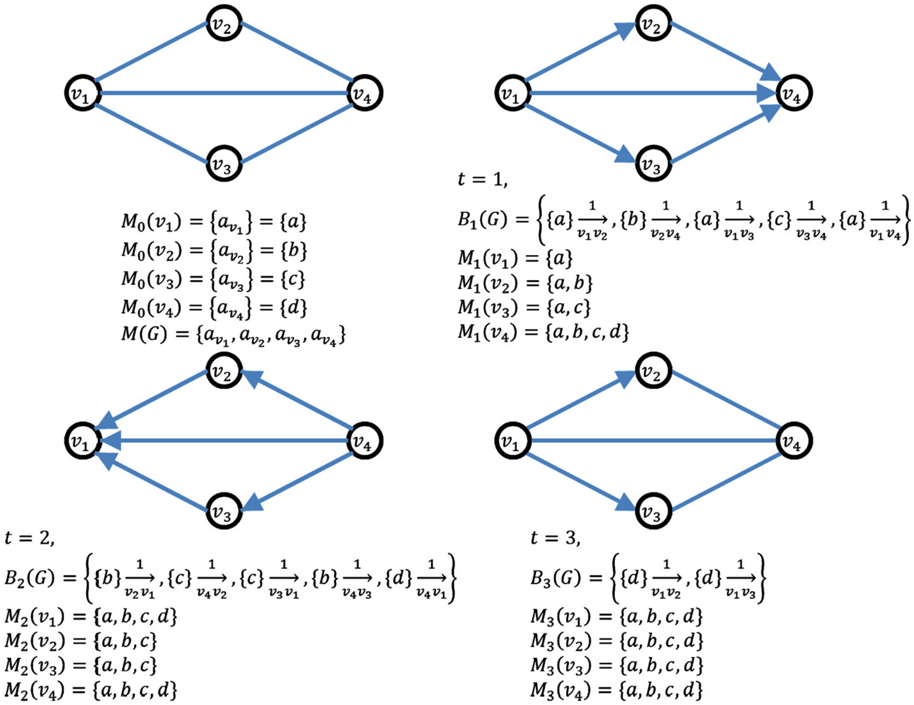

Fig. 1. 3-complete broadcasting set of $K_4 - e$.

Chang et al. [2] introduced the following information dissemination problem: Suppose each vertex has a private message needed to send to every other vertices (*all-to-all broadcast*). At each time unit, vertices exchange their messages under the following constraints:

(1) communication links are bidirectional and *half duplex* (that is, only one message can travel a link at a time unit),
(2) a message requires one time unit to be transferred between two nodes,
(3) each vertex can use all of its links at the same time (*all-port*).

Note that in this model, a vertex can send and receive messages during the same time unit, and a vertex can receive multiple messages during the same time unit. In [2], Chang et al. gave upper and lower bounds for the all-to-all broadcast number (the shortest time needed to complete the all-to-all broadcast) of graphs and gave formulas for the all-to-all broadcast number of trees, complete bipartite graphs and double loop networks under this model.

We study the *all-to-all broadcast* problem of Cartesian product of graphs in this paper. The model we consider is the same as that was given in [2]. We give upper and lower bounds for the all-to-all broadcast number of Cartesian product of graphs and give formulas for the all-to-all broadcast numbers of some classes of graphs, such as the Cartesian product of two cycles, the Cartesian product of a cycle with a complete graph of odd order, the Cartesian product of two complete graphs of odd order, and the hypercube Q_{2n}.

2. Preliminary

We first fix some notations that will be used later. From now on, when consider a graph G, we always assume that G is connected. Given a graph G and a vertex v in $V(G)$, we use $M_0(v) = \{a_v\}$ to denote that at the beginning, v owns a private message a_v. And we use $\{b\}^i_{\overrightarrow{uv}}$ to denote that at the ith time unit, u send the message b to vertex v, and call $\{b\}^i_{\overrightarrow{uv}}$ a *call*. A set of calls $B(G)$ is a *broadcasting set of* G if for each $\{b\}^i_{\overrightarrow{uv}} \in B(G)$, $b \in \{\, c : \{c\}^l_{\overrightarrow{wu}} \in B(G),\ 1 \le l \le i-1\} \cup \{a_u\}$. For a broadcasting set $B(G)$ of G, we use $\Delta_{B(G)}$ to denote the maximum number of time used in $B(G)$ (that is, $\Delta_{B(G)} = \max\{i : \{a\}^i_{\overrightarrow{uv}} \in B(G)\}$), and for all v in $V(G)$ and all i, $1 \le i \le \Delta_{B(G)}$, we use $(M_i(v))_{B(G)}$ (or, simply, $M_i(v)$, if $B(G)$ need not to be specified) to denote the set of messages received by vertex v till time unit i (that is, $(M_i(v))_{B(G)} = \{a : \{a\}^l_{\overrightarrow{uv}} \in B(G),\ 1 \le l \le i\} \cup \{a_v\}$). (Fig. 1 is an example for these notations.)

A broadcasting set $B(G)$ of G is a *complete broadcasting set of* G if $(M_{\Delta_{B(G)}}(v))_{B(G)} = \bigcup_{w \in V(G)} \{a_w\}$ for all $v \in V(G)$ (that is, if all vertices receive all the messages in $\bigcup_{w \in V(G)} \{a_w\}$ by using the broadcasting set $B(G)$). A complete broadcasting set of G is a *k-complete broadcasting set of* G if $\Delta_{B(G)} = k$. For a graph G, the *all-to-all broadcast number* of G, denoted by $t(G)$, is the shortest time needed to complete the broadcast; that is, to let all the vertices receive all the messages in