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Good spanning trees in graph drawing

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ABSTRACT

A plane graph is a planar graph with a fixed planar embedding. In this paper we define a special spanning tree of a plane graph which we call a good spanning tree. Not every plane graph has a good spanning tree. We show that every connected planar graph has a planar embedding with a good spanning tree. Using a good spanning tree, we show that every connected planar graph G of n vertices has a straight-line monotone grid drawing on an $O(n) \times O(n^2)$ grid, and such a drawing can be found in $O(n)$ time. Our results solve two open problems on monotone drawings of planar graphs posed by Angelini et al. Using good spanning trees, we also give simple linear-time algorithms for finding a 2-visibility representation of a connected planar graph G of n vertices on a $(2n-1) \times (2n-1)$ grid and for finding a spike-*VPG* representation of G on a $(2n-1) \times n$ grid.

1. Introduction

The field of graph drawing has been flourishing very much in the last two decades. Recent progress in computational geometry, topological graph theory, and order theory has considerably affected the evolution of this field, and has widened the range of issues being investigated. Drawing of planar graphs with various constraints imposed by application areas has been studied in recent years. Many algorithmic tools such as canonical ordering [1], regular edge labeling [2], Schnyder realizer [3], orderly spanning trees [4,5] have been developed to solve various types of graph drawing problems. In this paper we introduce a special spanning tree in a plane graph which we call a *good spanning tree*. A good spanning tree is an ordered rooted spanning tree of a plane graph where the tree edges and the non-tree edges incident to a vertex obey some properties. Fig. 1 illustrates a good spanning tree T where the tree edges are drawn as thick lines. Observe the ordering of the tree edges and non-tree edges incident to vertex v in T where the two sets of consecutive non-tree edges are separated by a set of consecutive tree edges and the tree edge (v, u). Also no non-tree edge incident to v has the other end on the path from r to v in T. A good spanning tree of a plane graph can be considered as a generalization of a Schnyder realizer of a triangulated plane graph. We give a formal definition of a good spanning tree in Section 2. Not every plane graph has a good spanning tree: for example, the plane graphs in Fig. 2 do not have good spanning trees. However, we show that every planar graph has a planar embedding with a good spanning tree. Furthermore, we show that a good spanning tree has useful applications in the field of graph drawing, namely, in monotone drawings, in 2-visibility representations and in *VPG* representations. We next discuss each of the representations briefly and present our result for each case.

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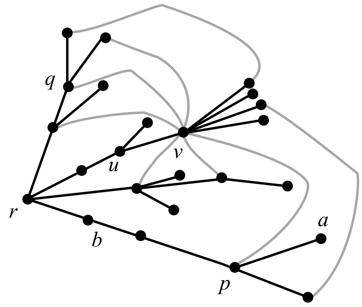

Fig. 1. Example of a good spanning tree T.

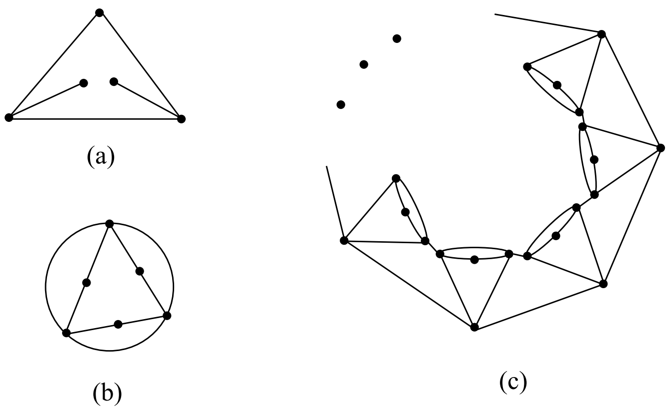

Fig. 2. Some examples of plane graphs that have no good spanning trees: (a) a connected plane graph, (b) a biconnected plane graph and (c) a plane graph with many vertices that has no good spanning tree.

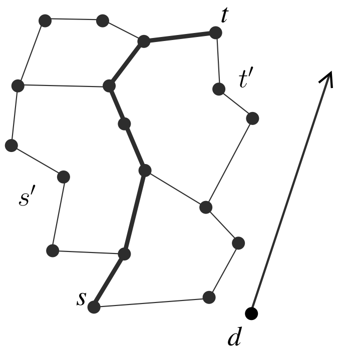

Fig. 3. The path between vertices s and t (as shown as a thick line) is monotone with respect to the direction d.

1.1. Monotone drawings

A *straight-line drawing* of a planar graph G is a drawing of G in which each vertex is drawn as a point and each edge is drawn as a straight-line segment without any edge crossing. A path P in a straight-line drawing of a planar graph is *monotone* if there exists a line l such that the orthogonal projections of the vertices of P on l appear along l in the order induced by P. A straight-line drawing Γ of a planar graph G is a *monotone drawing* of G if Γ contains at least one monotone path between every pair of vertices [6–8]. In the drawing of a graph in Fig. 3, the path between the vertices s and t drawn as a thick line is a monotone path with respect to the direction d, whereas no monotone path exists with respect to any direction between the vertices s' and t'. We call a monotone drawing of a planar graph a *monotone grid drawing* if every vertex is drawn on a grid point.

Monotone drawings of graphs are well motivated by human subject experiments by Huang et al. [9], who showed that the "geodesic tendency" (paths following a given direction) is important in comprehending the underlying graph. *Upward drawings* [10–13] are related to monotone drawings where every directed path is monotone with respect to the vertical line, while in a monotone drawing each monotone path, in general, is allowed to be monotone with respect to a different line.