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Inefficiency of equilibria for scheduling game with machine activation costs ☆

Ling Lin [a,b], Xiaochen Xian [b], Yujie Yan [b], Xing He [b], Zhiyi Tan [b,*,1]

[a] School of Computer & Computing Science, Zhejiang University City College, Hangzhou 310015, PR China
[b] Department of Mathematics, Zhejiang University, Hangzhou 310027, PR China

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ABSTRACT

In this paper, we study the scheduling game with machine activation costs. A set of jobs is to be processed on parallel identical machines. The number of machines available is unlimited, and an activation cost is needed whenever a machine is activated in order to process jobs. Each job chooses a machine on which it wants to be processed. The cost of a job is the sum of the load of the machine it chooses and its shared activated cost. The social cost is the total cost of all jobs. Representing the Price of Anarchy (PoA) and Price of Stability (PoS) as functions of the number of jobs, we get the tight bounds of PoA and PoS. Representing PoA and PoS as functions of the smallest processing time of jobs, asymptotically tight bound of PoA and improved lower and upper bounds of PoS are also given.

1. Introduction

In this paper, we study the scheduling game with machine activation costs. There is a set $\mathcal{J} = \{J_1, J_2, \cdots, J_n\}$ of jobs to be processed on parallel identical machines. The processing time of J_j is p_j, $j = 1, \cdots, n$. The number of machines available is unlimited, and an activation cost B is needed whenever a machine is activated in order to process jobs. Each job chooses a machine on which it wants to be processed. The choices of all jobs determine a schedule. The *load* of a machine M_i in a schedule is the sum of the processing time of all jobs selecting M_i. The activation cost of an activated machine is shared by the jobs selecting M_i, and the amount of each job shares is proportional to its processing time. The cost of a job in the schedule is the sum of the load of the machine it chooses and its shared activation cost. A schedule is a *Nash Equilibrium* (NE) if no job can reduce its cost by either moving to a different machine, or activating a new machine. The game model was first proposed by [9], and it was proved that an NE always exists for any job set $\mathcal{J}$.

Though the behavior of each job is influenced by individual costs, the performance of the whole system is measured by certain social cost. It is well known that in most situation NE are not optimal from this perspective due to lack of coordination. The inefficiency of NE can be measured by the *Price of Anarchy* (PoA for short) and *Price of Stability* (PoS for short) [11,1]. The PoA (PoS) of a job set is defined as the ratio between the maximal (minimal) social cost of an NE and an

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* Corresponding author.
E-mail address: tanzy@zju.edu.cn (Z. Tan).

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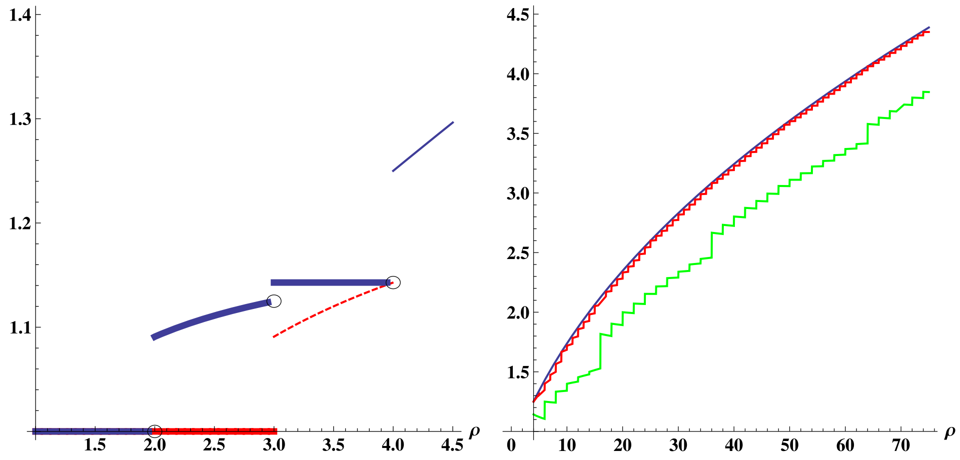

Fig. 1. The PoA and PoS of scheduling game with machine activation cost as a function of ρ. Left: $1 \le \rho \le 4$ (Solid lines represent PoA and dashed line represents PoS. Thick lines represent tight bounds. Thin lines represent upper bound of PoA and lower bound of PoS) Right: $\rho \ge 4$ (From top to bottom, upper bound of PoA, lower bound of PoA, lower bound of PoS).

optimal social cost. The PoA (PoS) of the game is the supremum value of the PoA (PoS) of all job sets. Clearly, $PoS \le PoA$ by definition.

The most favorite utilitarian social cost is the total cost of all jobs. Unfortunately, it is easy to show that the PoA of above game is infinite [3]. However, since PoA of the game is a kind of worst-case measure, it does not imply that the NE behaviors poorly for each job set. A common method to reveal the complete characteristic of NE in such situation is as follows: select a parameter and represent the PoA as a function of it. In [3], Chen and Gurel regarded the PoA as a function of $\rho = \frac{B}{\min_{1\le j\le n} p_j}$. Then they proved that the PoS is at least $\frac{\sqrt{\rho}+2}{4}$, and the PoA is at most $\frac{\rho+1}{2}$. However, the bounds are not tight.

Scheduling games with machine activation costs with different social costs were also studied in the literature. For the egalitarian social cost of minimizing the maximum cost among all jobs, Feldman and Tamir [9] proved that the PoA is $\frac{\tau+1}{2\sqrt{\tau}}$ when $\tau > 1$ and 1 when $0 < \tau \le 1$, where $\tau = \frac{B}{\max_{1\le j\le n} p_j}$. The PoS is $\frac{5}{4}$, and the bound is tight. Fruitful results on scheduling games without machine activation costs can be found in [11,8,2,4,10,5,7]. Apart from scheduling, results on various cost-sharing game in network routing and design can be found in [1,6].

In this paper, we revisit the scheduling game with machine activation cost with social cost of minimizing the total cost of jobs. Representing the PoA and PoS as functions of n, the number of jobs, we show that the PoA and PoS are both $\frac{n+1}{3}$, and the bound is tight. Representing the PoA and PoS as functions of ρ, we obtain asymptotically tight bound of PoA and improved lower bound of PoS. For small values of ρ, tight bounds of PoA and PoS are obtained. (Ref. Fig. 1). In more detail, the PoA is at most

$$\begin{cases} 1, & 1 \le \rho < 2, \\ \frac{6\rho}{5\rho+2}, & 2 \le \rho < 3, \\ \frac{8}{7}, & 3 \le \rho < 4, \\ \frac{\rho+1}{2\sqrt{\rho}}, & \rho \ge 4, \end{cases}$$

and the bound is tight when $1 \le \rho \le 4$ and ρ is a square of an integer. The PoS is 1 when $1 \le \rho \le 3$, at least $\frac{4\rho}{3\rho+2}$ when $3 < \rho < 4$. When $\rho \ge 4$, lower bound of PoS is also given, and it equals to $\frac{\rho+4}{2\sqrt{\rho}+3}$ when ρ is a square of an even number.

The paper is organized as follows. In Section 2, we give some notations and properties of both NE and optimal schedule. In Sections 3, we present the PoA as a function of the number of jobs. In Sections 4 and 5, we present the PoA and PoS as functions of the smallest processing time of the jobs, respectively.

2. Preliminaries

2.1. Problem and properties

Let $\mathcal{J} = \{J_1, J_2, \cdots, J_n\}$ be a job set. W.l.o.g., we assume $n \ge 2$ and $p_1 \ge p_2 \ge \cdots \ge p_n$. By scaling the processing times we can assume that $B = 1$. Denote $P = \sum_{j=1}^{n} p_j$. Write $\rho = \frac{1}{p_n}$ and $\tau = \frac{1}{p_1}$ for simplicity. Given a schedule σ^A, the number of machines activated in σ^A is denoted m^A. Denote by $\mathcal{J}_i^A$ the set of jobs processing on M_i, $i = 1, \cdots, m^A$. The number of jobs and the total processing time of jobs of $\mathcal{J}_i^A$ is denoted n_i^A and L_i^A, respectively. Let $n_{min}^A = \min_{1\le i\le m^A} n_i^A$ and $n_{max}^A =$