Theoretical Computer Science 606 (2015) 98–120

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Subexponential concurrent constraint programming

Carlos Olarte [a,*], Elaine Pimentel [a,*], Vivek Nigam [b,*]

[a] Universidade Federal do Rio Grande do Norte, Natal, Brazil
[b] Universidade Federal da Paraíba, João Pessoa, Brazil

ARTICLE INFO

Article history:
Received 8 September 2014
Received in revised form 3 March 2015
Accepted 15 June 2015
Available online 19 June 2015

Keywords:
Linear logic
Concurrent constraint programming
Proof systems

ABSTRACT

In previous works we have shown that linear logic with subexponentials (SELL), a refinement of linear logic, can be used to specify emergent features of concurrent constraint programming (CCP) languages, such as preferences and spatial, epistemic and temporal modalities. In order to do so, we introduced a number of extensions to SELL, such as subexponential quantifiers for the specification of modalities, and more elaborated subexponential structures for the specification of preferences. These results provided clear proof theoretic foundations to existing systems. This paper goes in the opposite direction, answering positively the question: can the proof theory of linear logic with subexponentials contribute to the development of new CCP languages? We propose a CCP language with the following powerful features: 1) computational spaces where agents can tell and ask preferences (soft-constraints); 2) systems where spatial and temporal modalities can be combined; 3) shared spaces for communication that can be dynamically established; and 4) systems that can dynamically create nested spaces. In order to provide the proof theoretic foundations for such a language, we propose a unified logical framework ($SELLS^{\Cap}$) combining the extensions of linear logic with subexponentials mentioned above, and showing that this new framework has interesting proof theoretical properties such as cut-elimination and a sound and complete focused proof system.

1. Introduction

Logic and proof theory play an important role in the design of programming languages. In fact, new programming constructs have been proposed by following tight connections between programming languages and proof theory. For example, we investigated recently in [1] a proof theoretic specification of the concurrent constraint programming (CCP [2]) languages introduced in [3] that mention epistemic (eccp) and spatial (sccp) modalities. We used as underlying logical framework linear logic with subexponentials (SELL) [4,5], together with new quantifiers on subexponentials: $\Cap$ and $\Cup$, allowing, respectively, the universal and existential quantification of subexponentials. The focusing discipline [12] then enforced that the obtained encodings for eccp and sccp are *faithful* w.r.t. CCP's operational semantics in a strong sense: one operational step matches exactly one logical phase. This is the strongest level of adequacy called adequacy on the level of derivations [6].

The study done in [1] allowed the development of extensions of eccp and sccp with features not available in [3]. For example, we were able to specify systems with an unbounded number of agents for eccp or spaces for sccp as well as to specify new constructs that allow the communication of location names [7].

* Corresponding authors.
E-mail addresses: carlos.olarte@gmail.com (C. Olarte), elaine.pimentel@gmail.com (E. Pimentel), vivek.nigam@gmail.com (V. Nigam).

http://dx.doi.org/10.1016/j.tcs.2015.06.031
0304-3975/

More recently, in [8], we have shown that SELL (without the subexponential quantifiers) can be configured to capture CCP languages manipulating soft-constraints [9]. The underlying (soft) constraint system in these CCP calculi is based on semirings structures and has been used for the specification of systems that mention preferences, e.g., costs, probabilities, levels of uncertainty (fuzzy information), etc. However, moving from hard (crisp) constraints to soft constraints was not followed by a corresponding logical/proof theoretic characterization of these systems. This is unfortunate because one of the key motivations of the original CCP was its tight connection to logic and proof theory which enabled the proposal of more advanced systems such as its linear version `lcc` [10]. Hence, the main contribution of [8] was to recover this connection by studying the proof theory of soft constraint systems in the form of SELL theories.

It is worthy saying that it is not possible to specify in SELL some notions of soft-constraints, namely those based on non-idempotent semirings. For this, we introduced in [8] a new proof system, called SELLS, for which the subexponential promotion rule behaves differently. This new rule is quite interesting since it coincides with the usual one in SELL in the case of idempotent semirings. Moreover, it faithfully captures notions such as probabilities and costs that require non-idempotent semirings as the underlying algebraic structure.

This tight correspondence between soft constraints and proof theory allows us now to extend `eccp` and `sccp` with soft constraints, thus strengthening the main results in [1] and [8]. Hence, SELL can be regarded as a uniform underlying logical framework for a number of programming languages.

Building the foundations of programming languages, such as CCP, on solid proof theory reduces the principles used for designing a programming language to the foundations of logic. The comparison of different programming principles and languages can then be established by soundness and completeness results. In contrast, while model-theoretic approaches have also played an important role in the development of programming languages, including CCP, they allow for many specifications, which are many times hard to be compared.

This paper continues our research program of using extensions of linear logic with subexponentials to provide solid proof-theoretical foundations to different CCP languages as well as for the development of new programming constructs. Our main goal is to propose a unified general logical framework where many variants of CCP may be specified, including new ones, all with clear proof-theoretical foundations.

We summarize our main contributions below:

- We propose new subexponential quantifiers which are considerably more expressive than the ones introduced in our previous work [1]. Subexponentials are organized into a pre-order specifying the provability relation between them. While in our previous work we allowed only the quantification of subexponentials that are in the ideal of a single subexponential, our new quantifiers allow for the quantification of subexponentials that appear in the ideal of any subexponential of a given non-empty set of subexponentials, or between two subexponentials. We prove that the resulting system, called $\text{SELLS}^{\Cap}$, admits cut-elimination;
- We demonstrate that a number of CCP languages with different modalities can be specified as $\text{SELLS}^{\Cap}$ theories. For that, we define a general language called $\mathcal{M}\texttt{ccp}$ where the programmer can express, and combine, different modalities. More precisely, we show how the new quantifiers naturally induce new CCP operators which allow for the sharing and exporting of information between processes. For instance, we show how to formally express the situation when some information can be exported from an agent a to another agent b, but it is confined to these agents only. Thus, two agents are able to share private spaces. Moreover, we allow the combination of spatial modalities, as proposed in [3], and preferences (soft-constraints) [11]. This means that agents may not only share constraints, but also preferences, allowing for the specification of systems with spatial modalities constrained to levels of uncertainty.
- Finally, we propose a focused proof system [12] for $\text{SELLS}^{\Cap}$. Focusing is a discipline on proofs introduced originally for linear logic in order to reduce the proof search non-determinism. We show that our focused proof system is complete with respect to $\text{SELLS}^{\Cap}$. Moreover, the adequacy results relating the operational semantics of $\mathcal{M}\texttt{ccp}$ and derivations in $\text{SELLS}^{\Cap}$ rely on the focused proof system.

The remainder of the paper is organized as follows. In Section 2, we review linear logic with subexponentials and propose $\text{SELLS}^{\Cap}$, which includes the new subexponential quantifiers. In this section we also prove that the system admits cut-elimination. Section 3 proposes a focused proof system for $\text{SELLS}^{\Cap}$ and prove its soundness and completeness. We use $\text{SELLS}^{\Cap}$ as a logical framework to propose new CCP constructs in Section 4 demonstrating that they increase considerably the expressiveness of existing CCP calculi. Finally, in Section 5, we conclude by commenting on related work and pointing out future work.

It should be noted that a preliminary attempt to using SELL in order to specify the behaviors described in this paper (see Section 4.4) was published in [7]. In this paper we give many more examples and explanations. We refine several technical details and present more detailed proofs. We add much more proof-theoretical machinery in order to specify new CCP constructs and we also give several examples on the relation between CCP and the $\text{SELLS}^{\Cap}$ proof theory. The new contributions with respect to [7] are: (1) we show how to combine, in a unique logical framework, spatial modalities and preferences. For that, (2) we develop the (focused) $\text{SELLS}^{\Cap}$ system; (3) the new type system for location introduced in Section 2.2 allows us to define in a neater way the generation of new locations to be shared among agents; finally, (4) we develop the theory of agents that can export information to sublocations, a feature considered neither in [3] nor in [7].