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Spread of influence in weighted networks under time and budget constraints [☆]

Ferdinando Cicalese [a], Gennaro Cordasco [b,*], Luisa Gargano [a], Martin Milanič [c], Joseph Peters [d], Ugo Vaccaro [a]

[a] Dept. of Computer Science, University of Salerno, Italy
[b] Dept. of Psychology, Second University of Naples, Viale Ellittico, Caserta 81100, Italy
[c] University of Primorska, UP IAM and UP FAMNIT, SI 6000 Koper, Slovenia
[d] School of Computing Science, Simon Fraser University, Canada

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ABSTRACT

Given a network represented by a weighted directed graph G, we consider the problem of finding a bounded cost set of nodes S such that the influence spreading from S in G, within a given time bound, is as large as possible. The dynamics that governs the spread of influence is the following: initially only elements in S are influenced; subsequently at each round, the set of influenced elements is augmented by all nodes in the network that have a sufficiently large number of already influenced neighbors. We prove that the problem is NP-hard, even in simple networks like complete graphs and trees. We also derive a series of positive results. We present exact pseudo-polynomial time algorithms for general trees, that become polynomial time in case the trees are unweighted. This last result improves on previously published results. We also design polynomial time algorithms for general weighted paths and cycles, and for unweighted complete graphs.

1. Introduction

1.1. Motivation

Social influence is the process by which individuals adjust their opinions, revise their beliefs, or change their behaviors as a result of interactions with other people. When exposed to the opinions of peers on a given issue, people tend to filter and integrate the information they receive and adapt their own judgements accordingly (see for instance [44]). This human tendency to harmonize their own ideas and customs with the opinions and behaviors of others [3] may occur for several reasons: a) the basic human need to be liked and accepted by others [5]; b) the belief that others, especially a majority group, have more accurate and trustworthy information than the individual [41]; c) the "direct-benefit" effect, implying

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[*] Corresponding author.

E-mail addresses: cicalese@dia.unisa.it (F. Cicalese), gennaro.cordasco@unina2.it (G. Cordasco), lg@dia.unisa.it (L. Gargano), martin.milanic@upr.si (M. Milanič), peters@cs.sfu.ca (J. Peters), uv@dia.unisa.it (U. Vaccaro).

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that an individual obtains an explicit benefit when he/she aligns his/her behavior with the behavior of others (e.g., [24], Ch. 17). It has not escaped the attention of advertisers[1] that the natural human tendency to conform can be exploited in *viral marketing* [33]. Viral marketing refers to the spread of information about products and behaviors, and their adoption by people. According to Lately [32], "*the traditional broadcast model of advertising-one-way, one-to-many, read-only is increasingly being superseded by a vision of marketing that wants, and expects, consumers to spread the word themselves*". For what strictly concerns us, the intent of maximizing the spread of viral information across a network naturally suggests many interesting optimization problems. Some of them were first articulated in the seminal papers [30,31], under various adoption paradigms. The recent monograph [13] contains an excellent description of the area. In the next section, we will explain and motivate our model of information diffusion, state the problem that we are investigating, describe our results, and discuss how they relate to the existing literature.

1.2. The model

Let $G = (V, E)$ be a directed graph, $c : V \to \mathbb{N} = \{1, 2, \ldots\}$ be a function assigning costs to vertices and $w : E \to \mathbb{N}_0 = \{0, 1, 2, \ldots\}$ be a function assigning weights to edges. The value $c(v)$ of each vertex $v \in V$ is a measure of how much it costs to initially convince the member v of the network to endorse a given product/behavior. The weight of an arc $e = (u, v) \in E$, denoted either by $w(e)$ or by $w(u, v)$, represents the amount of influence that node u exercises on node v. Let $t : V \to \mathbb{N}_0$ be a function assigning thresholds to the vertices of G. For each node $v \in V$, the threshold value $t(v)$ quantifies how hard it is to influence node v, in the sense that easy-to-influence elements of the network have "low" $t(\cdot)$ values, and hard-to-influence elements have "high" $t(\cdot)$ values [27].

A process of *influence diffusion* in G, starting at the subset of nodes $S \subseteq V$ (hereafter called a *target set*), is a sequence of vertex subsets

$$\mathsf{Influenced}[S,0] \subseteq \mathsf{Influenced}[S,1] \subseteq \ldots \subseteq \mathsf{Influenced}[S,\tau] \subseteq \ldots \subseteq V,$$

where $\mathsf{Influenced}[S,0] = S$, and such that for all $\tau > 0$,

$$\mathsf{Influenced}[S,\tau] = \mathsf{Influenced}[S,\tau-1] \cup \Big\{u : \sum_{v \in N^{in}(u) \cap \mathsf{Influenced}[S,\tau-1]} w(v,u) \ge t(u)\Big\}.$$

Here $N^{in}(u) = \{v : (v, u) \in E\}$ denotes the set of incoming neighbors of u, that is, the set of nodes in G having a directed arc towards u. In words, at each round τ a node u becomes influenced if the sum of the influences exercised on u by u's already influenced incoming neighbors meets or exceeds u's threshold $t(u)$. We say that node u is influenced *within* round τ if $u \in \mathsf{Influenced}[S,\tau]$; u is influenced *at* round $\tau > 0$ if $u \in \mathsf{Influenced}[S,\tau] \setminus \mathsf{Influenced}[S,\tau-1]$.

The problem that we introduce and study in this paper is defined as follows:

(λ, β)-MAXIMALLY INFLUENCING SET ((λ, β)-MIS).
Instance: A directed graph $G = (V, E)$, node thresholds $t : V \to \mathbb{N}_0$, vertex costs $c : V \to \mathbb{N}$, edge influences $w : E \to \mathbb{N}_0$, a latency bound $\lambda \in \mathbb{N}$ and a budget $\beta \in \mathbb{N}$.
Objective: Find a set $S \subseteq V$ such that $c(S) = \sum_{v \in S} c(v) \le \beta$ and $|\mathsf{Influenced}[S,\lambda]|$ is as large as possible.

Notice that the assumption that all vertex costs are positive is without loss of generality. Indeed, if $c(v) = 0$ for some vertex v in the graph, then we can consider a new graph G' obtained from G by eliminating v and by setting

$$t'(u) = \begin{cases} \max\{t(u) - w(v,u), 0\} & \text{if } u \text{ is an out-neighbor of } v \text{ in } G \\ t(u) & \text{otherwise.} \end{cases}$$

The decrease in the threshold of the neighbors of v implies that $\mathsf{Influenced}[S,\tau]$ in G' is equal to $\mathsf{Influenced}[S \cup \{v\},\tau]$ in G, for each $S \subseteq V - \{v\}$ and $\tau \ge 1$; hence S is an optimal solution for G' iff $S \cup \{v\}$ is an optimal solution for the original instance. The above transformation can be carried out for all vertices of zero cost in time $O(|V| + |E|)$ resulting in an equivalent instance in which all vertex costs are positive.

We are also marginally interested in the case in which the influence of each arc and the cost to initially activate each vertex are unitary (i.e., the network is unweighted), and the graph representing the network is symmetric, that is, $(u, v) \in E$ if and only if $(v, u) \in E$. In this particular scenario, studied in the conference version of this paper [19], the activation process obeys the following simpler rule: $\mathsf{Influenced}[S,0] = S$, and for all $\tau > 0$,

$$\mathsf{Influenced}[S,\tau] = \mathsf{Influenced}[S,\tau-1] \cup \Big\{u : \big|N(u) \cap \mathsf{Influenced}[S,\tau-1]\big| \ge t(u)\Big\},$$

and the question is to find a set of vertices S such that $|S| \le \beta$ and $|\mathsf{Influenced}[S,\lambda]|$ is as large as possible, where λ is given as input to the problem.

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[1] And politicians too [9,34,42,40].