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Single-machine batch scheduling of linear deteriorating jobs

Min Ji [a,*], Qinyun Yang [a], Danli Yao [a], T.C.E. Cheng [b]

[a] *School of Computer Science and Information Engineering, Contemporary Business and Trade Research Center, Zhejiang Gongshang University, Hangzhou 310018, PR China*
[b] *Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Kowloon, Hong Kong*

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ABSTRACT

We consider the problem of scheduling jobs in batches on a single machine where the processing time of each job is a simple increasing linear function of its waiting time, i.e., the time between the starting time of processing the batch to which the job belongs and the starting time of processing of the job. The objective is to minimize the sum of the total job flow time and the batching cost. We first show that the case with a given number of batches is strongly NP-hard and we present a fully polynomial-time approximation scheme (FPTAS) for this case. We then show that the case with an arbitrary number of batches is also strongly NP-hard and there is no polynomial-time approximation algorithm with a constant upper bound for this case unless $P = NP$.

1. Introduction

We consider scheduling of batches of linear deteriorating jobs on a single machine. The problem is motivated by the manufacturing of cold formed products in steel production as described in Low et al. [17], where the pertinent manufacturing operations include cold drawing, cold pressing, cold forming, and cold extrusion. Typically, different cold formed products are made in batches by rolling molten iron ingots, which are extracted from an electric furnace at a very high temperature and stored in a buffer awaiting the rolling process. Prior to rolling, an ingot needs to be preheated to reach a threshold temperature for the rolling operation. The preheating time of an ingot depends on its temperature at the beginning of the preheating process and the type of product to be made from it. The longer an ingot waits until the start of the preheating process, the more its temperature drops, so it takes a longer time to preheat it. Therefore, we can model the processing time of an ingot (i.e., the total preheating and rolling time) as an increasing linear function of its starting time. It follows that scheduling of steel production from molten iron ingots that involves the preheating process described above can in general be considered as single-machine batch scheduling of linear deteriorating jobs.

We formally describe the problem as follows: We are given a set of independent and non-preemptive deteriorating jobs $J = \{J_1, J_2, \dots, J_n\}$ to be scheduled and processed on a single machine. The machine can handle only one job at a time. The jobs are processed in batches on the machine. Each batch is preceded by a setup time $t_0 > 0$. Each job J_j ($j = 1, 2, \dots, n$) is associated with a deteriorating rate $\alpha_j > 0$. Let S_k and s_j denote the starting time of the kth batch to which job J_j belongs and the starting time of job J_j, respectively. The actual processing time of job J_j is a time-dependent function depending

* Corresponding author. Tel./fax: +86 571 28008303.
E-mail addresses: jimkeen@163.com (M. Ji), yangqinyun07@163.com (Q.Y. Yang), danli0911@126.com (D.L. Yao), Edwin.Cheng@polyu.edu.hk (T.C.E. Cheng).

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on the difference $s_j - S_k$, i.e., the actual processing time of job J_j is $p_j = \alpha_j(s_j - S_k)$. The completion time C_j of job J_j is the time when it finishes processing (i.e., "job availability", [21]), rather than the completion time of all the jobs in its batch (i.e., "batch availability", [21]). Thus, we denote the flow time of job J_j as $F_j = C_j - S_k$ if job J_j is processed in batch k. Our goal is to partition the jobs into batches, and sequence the batches and the jobs in each batch to minimize the sum of the total job flow time and the batching cost, i.e., $\sum F_j + \mu(B)$, where $\mu(B)$ is the batching cost, which is assumed to be a non-decreasing function of the number of batches B ($B \le n$) with $\mu(0) = 0$.

The above scheduling model comprises three broad categories of scheduling problems: (1) scheduling problems with time-dependent processing times, which was first studied by Melnikov and Shafransky [18]; (2) scheduling problems with setup times, which was reviewed by Allahverdi et al. [2]; and (3) batch scheduling problems, which was reviewed by Albers and Brucker [1]. Considering a closely related model with the objective of minimizing the makespan, Ji and Cheng [12] show that the problem is strongly NP-hard and present a fully polynomial-time approximation scheme (FPTAS) for the case where the number of batches B has a constant upper bound U, where $U \ge 2$.

Research on classical batch scheduling relevant to our model has been conducted by different researchers. Cheng et al. [5] present a single-machine scheduling problem with batch deliveries to minimize the sum of batch delivery and job earliness penalties. Wang and Cheng [23] study a batch delivery scheduling problem on m parallel machines where the jobs have constant processing times to minimize the sum of the total job flow time and delivery cost. Lee and Yoon [16] present a coordinated production and delivery scheduling problem that incorporates stage-dependent inventory holding costs. Extensive surveys on batch scheduling research can be found in Cheng and Gordon [4], Chang and Lee [3], Ji et al. [13], Qi [22], Yang et al. [24], and Morteza et al. [19]. However, all of these papers focus on batch delivery scheduling, i.e., they focus on the downstream of a supply chain. In practice, the upstream of a supply chain plays a crucial role in determining the efficiency of a supply chain. Therefore, in this paper we study batch scheduling that takes place in the upstream of a supply chain.

Given a partition of the jobs into B ordered batches and a job sequence for each batch, we denote by X_k the sequence of the jobs in the kth batch, $k = 1, 2, \ldots, B$, and by $|X_1|X_2|\ldots|X_B|$ the corresponding schedule. The goal is to find simultaneously the number of batches B, i.e., a partition of the jobs into batches, and the job sequence in each batch such that the sum of the total job flow time and the batching cost is minimized. Using the three-field notation of Graham et al. [7], we denote our scheduling problem as $1/bd, p_j = \alpha_j(s_j - S_k)/\sum F_j + \mu(B)$.

We organize the rest of the paper as follows: In Section 2 we show that the case $1/bd, p_j = \alpha_j(s_j - S_k), B \le U/\sum F_j + \mu(B)$ is NP-hard in the strong sense for any constant $U \ge 2$ and in Section 3 we present an FPTAS for this case. In Section 4 we show that the case with an arbitrary number of batches is also strongly NP-hard and in Section 5 we show that there is no polynomial-time approximation algorithm with a constant upper bound for this case. We conclude the paper and suggest some topics for future research in the final section.

2. NP-hardness of $1/bd, p_j = \alpha_j(s_j - S_k), B \le U/\sum F_j + \mu(B)$ for any constant $U \ge 2$

In this section we prove that the problem under study is NP-hard in the strong sense when the number of batches has a constant upper bound U for $U \ge 2$.

Theorem 1. *Problem $1/bd, p_j = \alpha_j(s_j - S_k), B \le U/\sum F_j + \mu(B)$ is NP-hard in the strong sense for any constant $U \ge 2$ even when $\mu(B)$ is a simple linear function of B.*

Proof. We accomplish this by performing a reduction from the Subset Product Problem, which Ng et al. [20] have shown to be NP-complete in the strong sense. An instance I of the Subset Product problem is formulated as follows:

Instance I. Given a finite set $X = \{1, 2, \ldots, k\}$, a size $x_j \in Z^+$ for each $j \in X$, and a positive integer A_1, does there exist a subset $X_1 \subseteq X$ such that the product of the size of the elements in X_1 satisfies $\prod_{j \in X_1} x_j = A_1$?

In the above instance, we can omit $j \in X$ with $x_j = 1$ because it will not affect the product of any subset. Thus, we can assume that $x_j \ge 2$ for every $j \in X$. In addition, we can assume that $A_2 = \prod_{j \in X} x_j / A_1$ is an integer, otherwise it can immediately be answered that there is no solution to the instance.

We set $D = \prod_{j \in X} x_j = A_1 A_2$ and $X_2 = X \backslash X_1$. Then $D \ge 2^k$ since every $x_j \ge 2$.

Instance II. Given instance I, we construct a corresponding instance of the decision version of the case $1/bd, p_j = \alpha_j(s_j - S_k), B \le U/\sum F_j + \mu(B)$ as follows:

- Number of jobs: $n = k + 4$.
- The batch setup time: $t_0 > 0$, which is arbitrary.
- Jobs' deteriorating rates: $\alpha_j = x_j - 1$, for $j = 1, 2, \ldots, k$; $\alpha_{k+1} = DA_1 - 1$; $\alpha_{k+2} = DA_2 - 1$; $\alpha_{k+3} = \alpha_{k+4} = D^3 - 1$.
- The upper bound on the batch number: $U = 2$.
- The batching cost: $\mu(B) = cB$, where $c = t_0 D^4$ is a constant.
- The threshold: $G = t_0(3D^4 + 2D^5)$.

Question. Is there a schedule under which the sum of the total job flow time and the batching cost is no more than G?