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Complexity of barrier coverage with relocatable sensors in the plane ☆

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ABSTRACT

We consider several variations of the problems of covering a set of barriers (modeled as line segments) using sensors that can detect any intruder crossing any of the barriers. Sensors are initially located in the plane and they can *relocate* to the barriers. We assume that each sensor can detect any intruder in a circular area of fixed range centered at the sensor. Given a set of barriers and a set of sensors located in the plane, we study three problems: (i) the feasibility of barrier coverage, (ii) the problem of minimizing the largest relocation distance of a sensor (MinMax), and (iii) the problem of minimizing the sum of relocation distances of sensors (MinSum). When sensors are permitted to move to arbitrary positions on the barrier, the MinMax problem is shown to be strongly NP-complete for sensors with arbitrary ranges. We also study the case when sensors are restricted to use *perpendicular* movement to one of the barriers. We show that when the barriers are parallel, both the MinMax and MinSum problems can be solved in polynomial time. In contrast, we show that even the feasibility problem is strongly NP-complete if two perpendicular barriers are to be covered, even if the sensors are located at integer positions, and have only two possible sensing ranges. On the other hand, we give an $O(n^{3/2})$ algorithm for a natural special case of this last problem.

1. Introduction

The protection of a region by sensors against intruders is an important application of sensor networks that has been previously studied in several papers. Each sensor is typically considered to be able to sense an intruder in a circular region

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of fixed range around the sensor. Previous work on region protection using sensors can be classified into two major classes. In the first body of work, called *area coverage*, the monitoring of an entire region is studied [13,16], and the presence of an intruder can be detected by a sensor anywhere in the region, either immediately after the appearance of an intruder, or within a fixed time delay. In the second body of work, called *barrier coverage*, a region is assumed to be protected by monitoring its perimeter, called the *barrier* [1,3,6,7,15], and an intruder is detected when crossing the barrier. Clearly, the second approach is less expensive in terms of the number of sensors required, and it is sufficient in many applications.

There are two different approaches to barrier coverage in the literature. In the first approach, a barrier is considered to be a narrow strip of fixed width. Sensors are dispersed randomly on the barrier, and the probability of barrier coverage is studied based on the density of dispersal. Since random dispersal may leave gaps in the coverage, some authors propose using several rounds of random dispersal for complete barrier coverage [10,20]. In the second approach, several papers assume that sensors, once dispersed, are mobile, and can be instructed to relocate from the initial position to a final position on the barrier in order to achieve complete coverage [2,6,7,9,14,17,18]. Clearly, when a sufficient number of sensors is used, this approach always guarantees complete coverage of the barrier. In order to minimize energy consumption by the mobile sensors, researchers have studied the problem of assigning final positions to the sensors that minimize some aspect of the relocation cost. The variations studied so far include centralized algorithms for minimizing the *maximum* relocation distance (*MinMax*) [6], the *sum* of relocation distances (*MinSum*) [7], or minimizing the *number* of sensors that relocate (*MinNum*) [17]; distributed algorithms for barrier coverage are considered in [9,18].

Circular barriers are studied in [4,19]. In [4], the authors considered moving n sensors to the perimeter of a given circular barrier to form a regular n-gon and provided an algorithm for MinMax problem that runs in $O(n^{3.5}\log n)$. In [19] the results of [4] for the MinMax problem are improved and an $O(n^{2.5}\log n)$ algorithm is given. Also, the authors introduced an algorithm that solves the MinSum problem when initial positions of sensors are on the perimeter of the circular barrier and runs in $O(n^4)$ time.

Most of the previous work on linear barriers is set in the one-dimensional setting: the barriers are assumed to be one or more line segments that are part of a line $\mathcal{L}$, and furthermore, the sensors are initially located on the same line $\mathcal{L}$. In [6] it was shown that there is an $O(n^2)$ algorithm for the MinMax problem in the case when the sensor ranges are identical. The authors also showed that the problem becomes NP-complete for sensors with arbitrary ranges if there are two barriers on $\mathcal{L}$. A polynomial time algorithm for the MinMax problem is given in [5] for arbitrary sensor ranges for the case of a single barrier, and an improved algorithm is given for the case when all sensor ranges are identical. In [7], it was shown that the MinSum problem is NP-complete when arbitrary sensor ranges are allowed, and an $O(n^2)$ algorithm is given when all sensing ranges are the same. Similarly as in the MinSum problem, the MinNum problem is NP-complete when arbitrary sensor ranges are allowed, and an $O(n^2)$ algorithm is given when all sensing ranges are the same [17].

In this paper we consider the algorithmic complexity of several natural generalizations of the barrier coverage problem with sensors of arbitrary ranges. We generalize the work in [5–7,17] in two significant ways. First, we assume that the initial positions of sensors are arbitrary points in the two-dimensional plane, not necessarily on the line containing the barrier. This assumption is justified since in many situations, initial dispersal of sensors on the line containing the barrier might not be possible. Second, we consider multiple barriers that are parallel or perpendicular to each other. This generalization is motivated by barrier coverage of the perimeter of an area.

1.1. Preliminaries and notation

Throughout the paper, we assume that we are given a set of sensors $S = \{s_1, s_2, \dots, s_n\}$ located in the plane in positions $p_1, p_2, \dots, p_n$, where $p_i = (x_i, y_i)$ for some real values x_i, y_i. The sensing ranges of the sensors are positive real values $r_1, r_2, \dots, r_n$, respectively. A sensor s_i can detect any intruder in the closed circular area around p_i of radius r_i. We assume that sensor s_i is mobile and thus can relocate itself from its initial location p_i to another specified location p'_i. A *barrier* b is a closed line segment in the plane. Given a set of barriers $\mathcal{B} = \{b_1, b_2, \dots, b_k\}$, and a set of sensors S of sensing ranges $r_1, r_2, \dots, r_n$ with $\sum_{i=1}^{n} 2r_i \geq \sum_{i=1}^{k} |b_i|$, initially located at positions $p_1, p_2, \dots, p_n$ in the plane, the *barrier coverage* problem is to determine for each i with $1 \leq i \leq n$, the final position of sensor s_i on one of the barriers denoted by p'_i, so that all barriers are collectively covered by the sensing ranges of the sensors. We call such an assignment of final positions a *covering assignment*. Fig. 1 shows an example of a barrier coverage problem and a possible covering assignment. Motivated by reducing the energy consumption of sensors, we are interested in optimizing some measure of the movement of sensors involved to achieve coverage, in particular MinMax and MinSum. We use standard cost measures such as Euclidean or rectilinear distance. The distance between the initial position p and a final position p' of a sensor is denoted by $d(p, p')$.

We are interested in the algorithmic complexity of three problems:

Feasibility problem: Given a set of sensors S located in the plane at positions $p_1, p_2, \dots, p_n$, and a set of barriers $\mathcal{B}$, determine if there exists a valid covering assignment, i.e. determine whether there exist final node positions $p'_1, p'_2, \dots, p'_n$ on the barriers such that all barriers in $\mathcal{B}$ are covered.

MinMax problem: Given a set of sensors S located in the plane at positions $p_1, p_2, \dots, p_n$, and a set of barriers $\mathcal{B}$, find final node positions $p'_1, p'_2, \dots, p'_n$ on the barriers so that all barriers in $\mathcal{B}$ are covered and $\max_{1 \leq i \leq n}\{d(p_i, p'_i)\}$ is minimized.