Theoretical Computer Science 579 (2015) 74–87

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Finding disjoint paths in networks with star shared risk link groups ☆,☆☆

Jean-Claude Bermond [a,b], David Coudert [b,a], Gianlorenzo D'Angelo [c], Fatima Zahra Moataz [a,b,*]

[a] Univ. Nice Sophia Antipolis, CNRS, I3S, UMR 7271, 06900 Sophia Antipolis, France
[b] Inria, France
[c] Gran Sasso Science Institute (GSSI), L'Aquila, Italy

ARTICLE INFO

Article history:
Received 30 January 2014
Received in revised form 9 February 2015
Accepted 10 February 2015
Available online 18 February 2015
Communicated by D. Peleg

Keywords:
Diverse routing
Shared Risk Link Group
Colored graph
Complexity
Algorithms
Disjoint paths

ABSTRACT

The notion of Shared Risk Link Groups (SRLG) has been introduced to capture survivability issues where some links of a network fail simultaneously. In this context, the *k-diverse routing* problem is to find a set of k pairwise SRLG-disjoint paths between a given pair of end nodes of the network. This problem has been proven NP-complete in general and some polynomial instances have been characterized.

In this paper, we investigate the k-diverse routing problem in networks where the SRLGs are localized and satisfy the *star property*. This property states that a link may be subject to several SRLGs, but all links subject to a given SRLG are incident to a common node. We first provide counterexamples to the polynomial time algorithm proposed by X. Luo and B. Wang (DRCN'05) for computing a pair of SRLG-disjoint paths in networks with SRLGs satisfying the star property, and then prove that this problem is in fact NP-complete. We then characterize instances that can be solved in polynomial time or are fixed parameter tractable, in particular when the number of SRLGs is constant, the maximum degree of the vertices is at most 4, and when the network is a directed acyclic graph.

Finally we consider the problem of finding the maximum number of SRLG-disjoint paths in networks with SRLGs satisfying the star property. We prove that this problem is NP-hard to approximate within $O(|V|^{1-\varepsilon})$ for any $0 < \varepsilon < 1$, where V is the set of nodes in the network. Then, we provide exact and approximation algorithms for relevant subcases.

1. Introduction

To ensure reliable communications in connection oriented networks such as optical backbone networks, many protection schemes have been proposed. One of the most used, called *dedicated path protection*, consists in computing for each demand both a working and a protection path. A general requirement is that these paths have to be disjoint, so that at least one

☆ Preliminary versions of this paper have been presented in [2,3].

☆☆ This work is partially supported by a grant from the "Conseil régional Provence Alpes-Côte d'Azur", ANR project Stint under reference ANR-13-BS02-0007, ANR program "Investments for the Future" under reference ANR-11-LABX-0031-01, and by European project FP7 EULER (Grant No. 258307), and the Research Grant 2010N5K7EB 'PRIN 2010' ARS TechnoMedia (Algoritmica per le Reti Sociali Tecno-mediate) from the Italian Ministry of University and Research.

* Corresponding author.
E-mail address: fatima-zahra.moataz@inria.fr (F.Z. Moataz).

http://dx.doi.org/10.1016/j.tcs.2015.02.012
0304-3975/

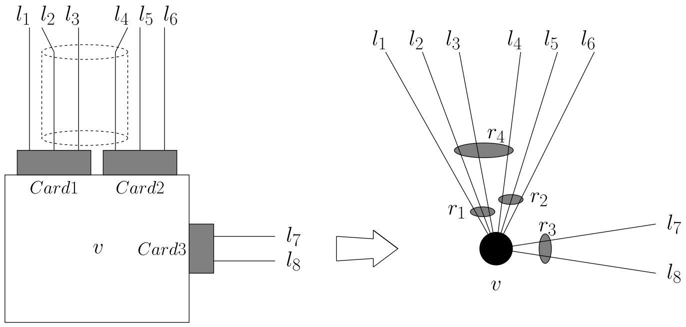

Fig. 1. Example of localized risks: link l_4 shares risk r_2, corresponding to Card 2 failure, with links l_5 and l_6, and shares risk r_4, corresponding to a conduit cut, with links l_2 and l_3.

of them can survive a single failure in the network. This method works well in a single link failure scenario, as it consists in finding two edge-disjoint paths between a pair of nodes. This is a well-known problem in graph theory for which there exist efficient polynomial time algorithms [29,30].

However, the problem of finding two disjoint paths between a pair of nodes becomes much more difficult, in terms of computational complexity, in case of multiple correlated link failures that can be captured by the notion of *Shared Risk Link Group* (or *SRLG*, for short). In fact, a SRLG is a set of network links that fail simultaneously when a given event (risk) occurs. The scope of this concept is very broad. It can correspond, for instance, to a set of fiber links of an optical backbone network that are physically buried at the same location and therefore could be cut simultaneously (i.e. backhoe or JCB fade). It can also represent links that are located in the same seismic area, or radio links in access and backhaul networks subject to localized environmental conditions affecting signal transmission, or traffic jam propagation in road networks. Note that a link can be affected by more than one risk. In practice, the failures are often localized and common SRLGs satisfy the *star property* [26] (coincident SRLGs in [12]). Under this property, all links of a given SRLG share an endpoint. Such failure scenarios can correspond to risks arising in router nodes like card failures or to the cut of a conduit containing links issued from a node (see Fig. 1).

The graph theoretic framework for studying optimization problems in networks with SRLGs is the *colored graph* model [9,11,12,15,26,34]. In this model, the network topology is modeled by a graph $G = (V, E)$ and the set of SRLGs by a set of colors $\mathcal{C}$. Each SRLG is modeled by a distinct color, and that color is assigned to all the edges corresponding to the network links subject to this SRLG. Also, an edge modeling a network link subject to several SRLGs will be assigned as many colors as SRLGs. A colored graph is therefore defined by the triple $(V, E, \mathscr{C})$, where $\mathscr{C}$ is a *coloring* function, $\mathscr{C} : E \rightarrow 2^{\mathcal{C}}$, that assigns a subset of colors to each edge. The colored graph model is also known as the *labeled graph* model [16]. Furthermore, some studies assumed that an edge is assigned at most one color [9,16,24]. Notice that the computational complexity of some optimization problems may be different in the model in which an edge is assigned at most one color than in the model in which it can be assigned multiple colors, and the impact of the transformation from one model to the other on problems complexities has been investigated in [10]. In the setting of colored graphs, the star property means that all the edges with a given color share a common vertex.

1.1. Related work

In the context of colored graphs, basic graph connectivity problems have been re-stated in terms of colors and proven much more difficult to address than their basic counterparts. For instance, the minimum color *st*-path problem is to find a path from vertex s to vertex t in the graph that minimizes the cardinality of the union of the colors of the edges along that path. This problem has been proven NP-hard [6,7,28] and hard to approximate [9,20] in general, $W[2]$-hard when parameterized by the number of used colors and $W[1]$-hard when parameterized by the number of edges of the path [16]. However, it has been proven in [10] that the minimum color *st*-path problem can be solved in polynomial time in colored graphs with the star property. Other optimization problems on graphs have been studied in the context of colored graphs such as the minimum color cut [9,15], the minimum color *st*-cut [9], the minimum color maximum matching [16].

The *k-diverse routing* problem in presence of SRLGs consists in finding a set of k SRLG-disjoint paths between a pair of vertices (i.e. paths having no risk in common). Note that many authors use, in the case $k = 2$, *diverse routing* instead of 2-diverse routing. With no restriction on the graph structure and on the assignment of SRLGs to edges, even finding two SRLG-disjoint paths is NP-complete [23], and therefore many heuristics have been proposed [19,28,32–35]. The problem is polynomial in some specific cases of localized failures: when SRLGs have span 1 (i.e. an edge can be affected by only one SRLG, and the set of edges belonging to the same SRLG forms a connected component, see [9]), and in a specific case of SRLGs having the star property [11] in which a link can be affected by at most two risks and two risks affecting the same link form stars at different nodes (this result also follows from [9]).