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Hardness results, approximation and exact algorithms for liar's domination problem in graphs

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ABSTRACT

A subset $L \subseteq V$ of a graph $G = (V, E)$ is called a *liar's dominating set* of G if (i) $|N_G[u] \cap L| \geq 2$ for every vertex $u \in V$, and (ii) $|(N_G[u] \cup N_G[v]) \cap L| \geq 3$ for every pair of distinct vertices $u, v \in V$. The Min Liar Dom Set problem is to find a liar's dominating set of minimum cardinality of a given graph G and the Decide Liar Dom Set problem is the decision version of the Min Liar Dom Set problem. The Decide Liar Dom Set problem is known to be NP-complete for general graphs. In this paper, we first present approximation algorithms and hardness of approximation results of the Min Liar Dom Set problem in general graphs, bounded degree graphs, and p-claw free graphs. We then show that the Decide Liar Dom Set problem is NP-complete for doubly chordal graphs and propose a linear time algorithm for computing a minimum liar's dominating set in block graphs.

1. Introduction

For a graph $G = (V, E)$, the sets $N_G(v) = \{u \in V | uv \in E\}$ and $N_G[v] = N_G(v) \cup \{v\}$ denote the *neighborhood* and the *closed neighborhood* of a vertex v, respectively. A set $D \subseteq V$ is a *dominating set* of G if for every vertex $v \in V \setminus D$, there exists $u \in D$ such that $uv \in E$. Equivalently, a subset D of V is a dominating set of G if $|N_G[v] \cap D| \geq 1$ for every $v \in V$. The *domination number* of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G. The concept of domination has been extensively studied, both in structural and algorithmic graph theory, because of its numerous applications to a variety of areas. Many variants of the basic concept of domination have appeared in literature due to different practical applications. For a comprehensive survey of this area, we refer to [13,14].

Among the different variants of domination in graphs, k-tuple domination is one of the widely studied variants. Formally, for a fixed integer $k > 0$, the *k-tuple dominating set* of a graph $G = (V, E)$ is a subset D_k of V such that $|N_G[v] \cap D_k| \geq k$ for every vertex $v \in V$. Note that k-tuple domination is a generalization of usual domination. The concept of k-tuple domination in graphs was introduced in [12]. The case when $k = 2$ is called *double domination* [12]. The *k-tuple domination number* of a graph G, denoted by $\gamma_{\times k}(G)$, is the minimum cardinality of a k-tuple dominating set of G. The k-tuple domination in graphs has been studied from algorithmic point of view in [16–18].

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Liar's domination in graphs [4,25,28] is a new and interesting variant of domination in graphs that has connection with both 2-tuple domination in graphs and 3-tuple domination in graphs. Liar's domination in graphs has been introduced in [4,28] due to the following situation in communication networks:

Assume that a graph $G = (V, E)$ models a communication network, where each vertex represents a communicating device and there is an edge between two vertices if the corresponding communicating devices can communicate to each other directly. Assume that each vertex of the graph G is the possible location for an *intruder* such as a thief, a saboteur, a fire or some possible fault. Assume also that there is exactly one intruder in the system represented by G. A protection device at a vertex v is assumed to be able to (i) detect the intruder at any vertex in its closed neighborhood $N_G[v]$, and (ii) report the vertex $u \in N_G[v]$ at which the intruder is located. One is interested in deploying protection devices at minimum number of vertices so that the intruder can be detected and identified correctly. This can be done by finding a minimum cardinality dominating set, say D, of G and deploying protection devices at all the vertices of D. If any protection device fails to detect the intruder, then to detect and identify the intruder correctly, one needs to place the protection devices at all the vertices of a minimum cardinality double-dominating set of G. However, any protection device in the closed neighborhood of the intruder vertex may (either deliberately or through a transmission error) misreport (lie) the location of an intruder in its closed neighborhood. Then to detect and identify the intruder correctly, placing the protection devices at all the vertices of a minimum double-dominating set does not work. To tackle this situation, the concept of *liar's domination* in graphs was introduced and the following theorem was proved in [28].

Theorem 1.1. *(See [28].) A set $L \subseteq V$ is a liar's dominating set of a graph $G = (V, E)$ if and only if (i) $|N_G[v] \cap L| \geq 2$ for every $v \in V$, and (ii) $|(N_G[u] \cup N_G[v]) \cap L| \geq 3$ for every pair of distinct vertices $u, v \in V$.*

Because of Theorem 1.1, throughout this paper we use the following as the definition of liar's dominating set in a graph.

Definition 1.2. A set $L \subseteq V$ of a graph $G = (V, E)$ is called a *liar's dominating set* if (i) for all $v \in V$, $|N_G[v] \cap L| \geq 2$, and (ii) for every pair $u, v \in V$ of distinct vertices, $|(N_G[u] \cup N_G[v]) \cap L| \geq 3$.

The *liar's domination number*, denoted by $\gamma_{LR}(G)$, is the minimum cardinality of a liar's dominating set of a graph G. The Min Liar Dom Set problem is to find a liar's dominating set of minimum cardinality of a given graph G and the Decide Liar Dom Set problem is the decision version of the Min Liar Dom Set problem. The Decide Liar Dom Set problem is known to be NP-complete for bipartite graphs [25], split graphs [21] and undirected path graphs [22]. The Min Liar Dom Set problem can be polynomially solved in trees [21] and proper interval graphs [22]. Some variations of liar's domination are also studied in the literature [20,23].

In this paper, we study the approximability of the Min Liar Dom Set problem. The main contributions are summarized below.

1. We show that for a graph $G = (V, E)$, the Min Liar Dom Set problem cannot be approximated within $(\frac{1}{8} - \varepsilon)\ln|V|$ for any $\varepsilon > 0$, unless $\mathsf{NP} \subseteq \mathsf{DTIME}(|V|^{O(\log\log|V|)})$.
2. We propose a polynomial time $3 + 2\ln(\Delta(G) + 1)$-factor approximation algorithm for the Min Liar Dom Set problem for general graphs, where $\Delta(G)$ is the maximum degree of the graph G.
3. We show that the Min Liar Dom Set problem is APX-complete for graphs with maximum degree 4.
4. We study the Min Liar Dom Set problem on p-claw free graphs. In particular, we first prove that the Decide Liar Dom Set problem is NP-complete for p-claw free graphs. We then propose a $2(p-1)$-factor approximation algorithm for the Min Liar Dom Set problem on p-claw free graphs. Finally, we show that the Min Liar Dom Set problem is APX-complete for p-claw free graphs for $p \geq 5$.
5. We study the Min Liar Dom Set problem on subclasses of chordal graphs. We first show that the Decide Liar Dom Set problem is NP-complete for doubly chordal graphs and then we propose a linear time algorithm for computing a minimum liar's dominating set in block graphs. Note that doubly chordal graphs and block graphs are two well known subclasses of chordal graphs [5].

The rest of the paper is organized as follows. In Section 2, we present some pertinent definitions and notations. In Section 3, we present a hardness result and an approximation algorithm for the Min Liar Dom Set problem for general graphs. In Section 4, we prove that the Min Liar Dom Set problem is APX-complete for graphs with maximum degree 4. In Section 5, we first prove that the Decide Liar Dom Set problem is NP-complete for p-claw free graphs. We then propose a $2(p-1)$-factor approximation algorithm for the Min Liar Dom Set problem on p-claw free graphs. Finally, we show that the Min Liar Dom Set problem is APX-complete for p-claw free graphs for $p \geq 5$. In Section 6, we show that Decide Liar Dom Set problem is NP-complete for doubly chordal graphs and then we propose a linear time algorithm for computing a minimum liar's dominating set in block graphs. Finally Section 7 concludes the paper.