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On the parameterized complexity of vertex cover and edge cover with connectivity constraints ☆

Henning Fernau [a], Fedor V. Fomin [b], Geevarghese Philip [d,*], Saket Saurabh [b,c]

[a] Universität Trier FB 4, Abteilung Informatik, 54286 Trier, Germany
[b] Department of Informatics, University of Bergen, 5020 Bergen, Norway
[c] The Institute of Mathematical Sciences, Chennai 600 113, India
[d] Max-Planck-Institut für Informatik, 66123 Saarbrücken, Germany

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ABSTRACT

We investigate the effect of certain natural connectivity constraints on the parameterized complexity of two fundamental graph covering problems, namely Vertex Cover and Edge Cover. Specifically, we impose the additional requirement that each connected component of a solution have at least t vertices (resp. edges from the solution) for a fixed positive integer t, and call the problem t-Total Vertex Cover (resp. t-Total Edge Cover). In both cases the parameter k is the size of the solution. We show that

- both problems remain fixed-parameter tractable with these restrictions, with running times of the form $\mathcal{O}^\star(c^k)$ for some constant $c > 0$ in each case, where the $\mathcal{O}^\star$ notation hides polynomial factors;
- for each fixed $t \geq 2$, t-Total Vertex Cover has no polynomial kernel unless CoNP $\subseteq$ NP/poly;
- for each fixed $t \geq 2$, t-Total Edge Cover has a linear vertex kernel of size $\frac{t+1}{t}k$.

These results significantly improve earlier work on these problems. We illustrate the utility of the technique used to solve t-Total Vertex Cover, by applying it to derive an $\mathcal{O}^\star(c^k)$-time FPT algorithm for the t-Total Edge Dominating Set problem.
Our no-poly-kernel result for t-Total Vertex Cover, and the known NP-hardness result for t-Total Edge Cover, are in stark contrast to the fact that Vertex Cover has a $2k$ vertex kernel, and that Edge Cover is solvable in polynomial time. This illustrates how even the slightest connectivity requirement results in a drastic change in the tractability of problems—the curse of connectivity!

☆ A preliminary version of this paper appeared in the proceedings of COCOON 2010 [1]. In addition to the detailed proofs which were omitted in that version, the current article also includes an FPT algorithm for the t-Total Edge Dominating Set problem which was not discussed in the shorter version.

* Corresponding author.
E-mail addresses: fernau@uni-trier.de (H. Fernau), fomin@ii.uib.no (F.V. Fomin), gphilip@mpi-inf.mpg.de (G. Philip), saket@imsc.res.in, saket@ii.uib.no (S. Saurabh).

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1. Introduction

Given a graph G and a positive integer k as input, the Vertex Cover problem asks whether G has a set of at most k vertices—a *vertex cover* of G—such that every edge of G has at least one of these vertices as an end-point. The Edge Cover problem is quite similar: given (G, k) as input, the question here is whether G has a set of at most k *edges*—an *edge cover* of G—such that every vertex in G is an end-point of at least one of these edges.

Vertex Cover is a classical NP-hard problem [2] whose parameterized version with k as the parameter is arguably the most investigated problem in parameterized algorithmics. The fastest known FPT algorithm for Vertex Cover runs in $\mathcal{O}^\star(1.2738^k)$ time [3], and the problem has a kernel with at most $2k$ vertices [4]. The parameterized complexity of Vertex Cover—with various parameterizations—continues to be actively investigated. Recent results of this nature include faster FPT algorithms for certain "above-guarantee" versions of Vertex Cover [5–7]. In contrast, the Edge Cover problem is solvable in polynomial time [8].

We investigate the parameterized complexity of variants of Vertex Cover and Edge Cover where additional connectivity constraints are imposed on the solution set S. More specifically, for each fixed integer t; $1 \leq t \leq k$ we define variants of the two problems, named t-Total Vertex Cover and t-Total Edge Cover, respectively, as follows.

t-Total Vertex Cover
Input: A graph $G = (V, E)$, and a non-negative integer k.
Parameter: k
Question: Does G have a vertex cover S of size at most k such that each connected component of the subgraph of G induced by S contains at least t vertices?

t-Total Edge Cover
Input: A graph $G = (V, E)$, and a non-negative integer k.
Parameter: k
Question: Does G have an edge cover T of size at most k such that each connected component of the subgraph of G induced by T contains at least t edges?

Observe that for $t = 1$, these problems are identical to Vertex Cover and Edge Cover, respectively. Observe also that if $k < t$ then the answer is trivially No for both the problems. Hence the interesting values of t are those in the range $1 \leq t \leq k$. A vertex cover satisfying the conditions specified in the first problem is called a *t-total vertex cover* of the graph G; an edge cover satisfying the conditions of the second problem is called a *t-total edge cover* of G.

The underlying classical problems were first investigated by Fernau and Manlove [9], who showed that t-Total Vertex Cover is NP-hard for each fixed $t \in \mathbb{N}$; $1 \leq t \leq k$, and that t-Total Edge Cover is NP-hard for each fixed $t \in \mathbb{N}$; $2 \leq t \leq k$. They also initiated the study of the parameterized variants of these problems.

Małafiejski and Żylinski studied 2-Total Edge Cover as a model of weak cooperation of guards in an art gallery problem [10]. Both Fernau and Manlove [9] and Małafiejski and Żylinski [10] derived a Gallai type identity which says that under certain conditions, the sum of (i) the cardinality of the largest possible packing of a graph with vertex-disjoint copies of a path of length two and (ii) the size of the smallest 2-total edge cover, equals the number of vertices of the graph. Fernau and Manlove also derived a generalization of this result to every fixed integer $t \geq 2$ [9]. Combining this with the result of Kirkpatrick and Hell [11], who proved that finding a packing of vertex-disjoint copies of trees on t edges in a graph is NP-hard, they showed that t-Total Edge Cover is NP-complete for each fixed $t \geq 2$.

Besides the art gallery problem mentioned above, further motivation for studying these problems can be drawn from certain models of fault-tolerant computing [12]. These problems are interesting from the point of view of computational biology as well, due to the close relation that these problems have to variants of the so-called Test Cover Problem [13].

Fernau and Manlove [9] derived a number of results for these problems. Apart from the NP-hardness results mentioned above, they showed that for each fixed $t \in \mathbb{N}$; $2 \leq t \leq k$ the t-Total Vertex Cover problem has a polynomial-time 2-approximation, but cannot be approximated within a factor of $10\sqrt{5} - 21 - \epsilon$ for any $\epsilon > 0$ in polynomial time unless $P = NP$. They further showed that for each fixed $t \in \mathbb{N}$; $2 \leq t \leq k$ the t-Total Edge Cover problem has a polynomial-time 2-approximation, and that there exists a constant $\delta > 1$ such that 2-Total Edge Cover cannot be approximated within a factor of δ in polynomial time unless $P = NP$. As for the parameterized versions of these problems, they showed that (i) 2-Total Vertex Cover is FPT and can be solved in $\mathcal{O}^\star(2.3655^k)$ time, and that (ii) for each fixed $t \in \mathbb{N}$; $2 \leq t \leq k$, t-Total Edge Cover is FPT and can be solved in $\mathcal{O}^\star((2k)^{2k})$ time. They also claimed to prove that the t-Total Vertex Cover problem has a