Theoretical Computer Science 565 (2015) 63–76

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

# Reductions between scheduling problems with non-renewable resources and knapsack problems

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ARTICLE INFO

*Article history:*
Received 7 December 2013
Received in revised form 11 September 2014
Accepted 6 November 2014
Available online 13 November 2014
Communicated by G. Ausiello

*Keywords:*
Approximation preserving reductions
Scheduling problems
Knapsack problems

ABSTRACT

In this paper we establish approximation preserving reductions between scheduling problems in which jobs either consume some raw materials, or produce some intermediate products, and variants of the knapsack problem. Through the reductions, we get new approximation algorithms, as well as inapproximability results for the scheduling problems.


## 1. Introduction

In this paper we study approximation preserving reductions between single machine scheduling problems extended with non-renewable resources, and various knapsack problems. We will consider two types of scheduling problems: (i) scheduling of jobs *producing* some intermediate products, and (ii) scheduling of jobs *consuming* some raw materials. In the former case, the jobs produce intermediate products to meet demands at given dates, whereas in the second case, jobs consume raw materials whose stock is replenished at given dates and in known quantities. On the other hand, we will consider two variants of the knapsack problem. Beside the *basic knapsack problem*, in which there is a set of items each having a size and a profit, and a subset of items of maximum profit, but of limited total size must be chosen, we will also consider the *multi-dimensional knapsack problem* in which the knapsack has sizes in multiple dimensions.

Approximation preserving reductions are useful for obtaining both positive and negative results. Consider, say, the PTAS reduction, which reduces an optimization problem $\Pi_1$ to another optimization problem $\Pi_2$ in such a manner that if there is a PTAS for $\Pi_2$, then this yields a PTAS for $\Pi_1$ as well (for formal definitions, see Section 3). So, we can get a positive result for an optimization problem $\Pi_1$, i.e., a PTAS, if we can identify another optimization problem $\Pi_2$ which admits a PTAS, and if we manage to devise a PTAS reduction from $\Pi_1$ to $\Pi_2$. On the other hand, if we want to prove that some problem $\Pi_2$ does not admit a PTAS unless $\mathcal{P} = \mathcal{NP}$, it suffices to find another optimization problem $\Pi_1$ which does not admit a PTAS unless $\mathcal{P} = \mathcal{NP}$, and a PTAS reduction from $\Pi_1$ to $\Pi_2$. Among the many types of reductions published in the literature, we will only use the PTAS-, the FPTAS- and the Strict-reductions (see Section 3).

Before we proceed we provide a more formal definition of those problems studied in this paper.

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http://dx.doi.org/10.1016/j.tcs.2014.11.007
0304-3975/

### 1.1. Knapsack problems

In the (basic) *Knapsack Problem* (*KP*) there is a set of $n$ items $j$ with profit $v_j$ and weight $w_j$. One has to select a subset of the items with the largest total profit so that the total weight of the selected items is at most a given constant ('capacity') $b'$. Formally:

$$OPT_{KP} := \max \sum_{j=1}^{n} v_j x_j \tag{1}$$

$$\sum_{j=1}^{n} w_j x_j \le b' \tag{2}$$

$$x_j \in \{0,1\}, \quad j = 1, \dots, n. \tag{3}$$

We will use the notation $OPT_{KP}$ for the optimal value of this problem.

In the *r-dimensional Knapsack Problem* (*r-DKP*) each item has $r$ weights and there are $r$ constraints:

$$OPT_{r-DKP} := \max \sum_{j=1}^{n} v_j x_j \tag{4}$$

$$\sum_{j=1}^{n} w_{ij} x_j \le b'_i, \quad i = 1, \dots, r \tag{5}$$

$$x_j \in \{0,1\}, \quad j = 1, \dots, n. \tag{6}$$

The optimum value of this problem is denoted by $OPT_{r-DKP}$.

### 1.2. Resource scheduling problems

In this section we recapitulate two resource scheduling problems, the *Delivery tardiness problem* (see [10]) and the *Material consumption problem* (see e.g. [5,15]).

In the *Delivery tardiness problem* ($DTP^r_q$) there are a single machine, a finite set of $n$ jobs, and a set of $r$ materials produced by the jobs. The machine can perform only one job at a time, and preemption is not allowed. Job $J_j$, $j \in \{1, \dots, n\}$, has a processing time $p_j \in \mathbb{Z}_+$, and produces some materials, which is described by an $r$-dimensional non-negative vector $a_j \in \mathbb{Z}^r_+$. There are due dates along with required shipments, i.e., pairs $(u_\ell, b_\ell)$ with $u_\ell \in \mathbb{Z}_+$, and $b_\ell \in \mathbb{Z}^r_+$, $\ell = 1, \dots, q$, and $0 \le u_1 < \dots < u_q$. The solution of the problem is a sequence $\sigma$ of the jobs. The starting time of the $i$th job is then $S_{\sigma(i)} = \sum_{k=1}^{i-1} p_{\sigma(k)}$. A shipment $(u_\ell, b_\ell)$ is *met* by $S$, if the total production of those jobs finishing by $u_\ell$ is at least $\tilde{b}_\ell := \sum_{k=1}^{\ell} b_k$, i.e., $\sum_{(j\,:\,S_j+p_j \le u_\ell)} a_j \ge \tilde{b}_\ell$ (coordinate wise), otherwise it is *tardy*. Let $C_\ell(S)$ be the earliest time point $t \ge 0$ with $\sum_{(j\,:\,S_j+p_j \le t)} a_j \ge \tilde{b}_\ell$. The *tardiness* of a shipment is $T_\ell(S) := \max\{0, C_\ell(S) - u_\ell\}$. The *maximum tardiness of a schedule* is $T_{\max}(S) := \max_\ell T_\ell(S)$. The objective is to minimize the maximum tardiness. We denote this problem by $1|dm = r|T_{\max}$, where '$dm = r$' indicates that the number of products is fixed to $r$ (not part of the input). An important special case of this problem is when there are only two time points ($0 \le u_1 < u_2$) when some product is due (denoted by $1|dm = r, q = 2|T_{\max}$). Since $T_{\max}$ can be 0 in an optimal solution, we will consider the *shifted delivery tardiness objective function* defined as $T^s_{\max} := T_{\max} + const$, where *const* is a positive constant, depending on the problem data.

In the *Material consumption problem* ($MCP^r_q$) there are a single machine, a finite set of $n$ jobs, and a set of $r$ materials consumed by the jobs. The machine can perform only one job at a time, and preemption is not allowed. There are $n$ jobs $J_j$, $j = 1, \dots, n$, each characterized by two numbers: processing time $p_j$ and quantities consumed from the resources $a_j \in \mathbb{Z}^r_+$. The resources have initial stocks, and they are replenished at given moments in time, i.e., there are $q$ pairs $(u_1, b_1), \dots, (u_q, b_q)$, with $0 = u_1 < \dots < u_q$ being the time points and the $b_\ell \in \mathbb{Z}^r_+$ the quantities supplied. A *schedule* $S$ specifies a starting time for each job such that the jobs do not overlap in time, and the total material supply up to the starting time of every job is at least the total request of those jobs starting not later than $S_j$, i.e., $\sum_{(\ell\,:\,u_\ell \le S_j)} b_\ell \ge \sum_{(j'\,:\,S'_j \le S_j)} a'_j$ (coordinate wise). The objective is to minimize the *makespan* defined as the maximum job completion time. We denote this problem by $1|rm = r|C_{\max}$, where '$rm = r$' indicates that the number of the raw materials is fixed to $r$ (not part of the input). An important special case of this problem is when there are only two time points ($u_1 = 0$ and $u_2 > 0$) when some resource is supplied ($1|rm = r, q = 2|C_{\max}$).

**Assumption 1.** *In both problems $\sum_\ell b_\ell = \sum_j a_j$ holds without loss of generality.*

The notation used throughout the paper is summarized in Appendix A.