Theoretical Computer Science 555 (2014) 71–84

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Base-object location problems for base-monotone regions

Jinhee Chun [a], Takashi Horiyama [b], Takehiro Ito [a], Natsuda Kaothanthong [a], Hirotaka Ono [c], Yota Otachi [d,*], Takeshi Tokuyama [a], Ryuhei Uehara [d], Takeaki Uno [e]

[a] Graduate School of Information Sciences, Tohoku University, Sendai 980-8579, Japan
[b] Graduate School of Science and Engineering, Saitama University, Saitama 338-8570, Japan
[c] Department of Economic Engineering, Kyushu University, 6-19-1 Hakozaki Higashi-ku, Fukuoka 812-8581, Japan
[d] School of Information Science, Japan Advanced Institute of Science and Technology, Asahidai 1-1, Nomi, Ishikawa 923-1292, Japan
[e] National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo, 101-8430, Japan

ARTICLE INFO

Available online 25 November 2013

Keywords:
Base-monotone region
Room-Edge Problem
Computational complexity
Image segmentation

ABSTRACT

A base-monotone region with a base is a subset of the cells in a pixel grid such that if a cell is contained in the region then so are the ones on a shortest path from the cell to the base. The problem of decomposing a pixel grid into disjoint base-monotone regions was first studied in the context of image segmentation. It is known that for a given pixel grid and base-lines, one can compute in polynomial time a maximum-weight region that can be decomposed into disjoint base-monotone regions with respect to the given base-lines (Chun et al., 2012 [4]). We continue this line of research and show the NP-hardness of the problem of optimally locating k base-lines in a given $n \times n$ pixel grid. We then present an $O(n^3)$-time 2-approximation algorithm for this problem. We also study two related problems, the k base-segment problem and the quad-decomposition problem, and present some complexity results for them.

1. Introduction

Let P be an $n \times n$ pixel grid. A *pixel* (i, j) of P is the unit square whose top-right corner is the grid point $(i, j) \in \mathbb{Z}^2$. For example the bottom-left cell of P is $(1, 1)$ and the top-right cell is (n, n). Each pixel $p = (i, j)$, where $1 \leqslant i, j \leqslant n$, has its *weight* $w(p) \in \mathbb{Z}$. Now we define the following general problem.

Problem: Maximum Weight Region Problem (MWRP)
Instance: An $n \times n$ pixel grid P.
Objective: Find a region $R \in \mathcal{F}$ maximizing the weight $w(R) = \sum_{p \in R} w(p)$, where $\mathcal{F} \subseteq 2^P$ is a family of pixel regions.

The general problem MWRP has been studied for several families $\mathcal{F}$ that are related to practical problems (see [2,4] and the references therein). Observe that if $\mathcal{F} = 2^P$, then R can be arbitrarily chosen, and thus the answer is the set of all positive cells. On the other hand, if $\mathcal{F}$ is the family of connected regions (in the usual 4-neighbor topology), then the

* Corresponding author.
E-mail addresses: jinhee@dais.is.tohoku.ac.jp (J. Chun), horiyama@al.ics.saitama-u.ac.jp (T. Horiyama), takehiro@ecei.tohoku.ac.jp (T. Ito), natsuda@dais.is.tohoku.ac.jp (N. Kaothanthong), hirotaka@en.kyushu-u.ac.jp (H. Ono), otachi@jaist.ac.jp (Y. Otachi), takeshi@dais.is.tohoku.ac.jp (T. Tokuyama), uehara@jaist.ac.jp (R. Uehara), uno@nii.ac.jp (T. Uno).

0304-3975/$ – see front matter
http://dx.doi.org/10.1016/j.tcs.2013.11.030

Fig. 1. Image segmentation via k base-line MWRP proposed in [4]. We first convert a picture to a gray scale image. Next, with some suitable function, we construct a pixel grid in which each dark pixel has positive weight and each light pixel has negative weight. Finally we solve the k base-line MWRP to segment the background from the objects. In this example, the boundary edges of the picture are used as base-lines (thus $k = 4$). For example, the red region in the third figure has the top edge of the image as its base-line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

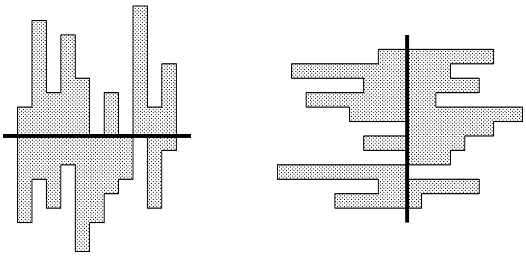

Fig. 2. A based x-monotone region (left) and a based y-monotone region (right).

Fig. 3. The complement of a base-monotone feasible region may represent an object in a picture nicely. By additional base-lines, the result may be improved.

problem becomes NP-hard [2]. For the complexity of MWRP for other families, see the paper by Chun et al. [4] and the references therein.

Motivated by the image segmentation problem, Chun et al. [4] studied a more complicated family of pixel regions for MWRP (see Fig. 1). A *base-line* of an $n \times n$ pixel grid P is a vertical line $x = b$ or horizontal line $y = b$, where $0 \leqslant b \leqslant n$. A pixel region R is a *based x-monotone region* if there is a horizontal base-line $y = b$ such that if a cell is contained in R, then so are the ones on a shortest path (in the usual 4-neighborhood graph) from the cell to the base. *Based y-monotone regions* are analogously defined. Based x-monotone regions and based y-monotone regions are *base-monotone regions* (Fig. 2). Given a set of k base-lines, a region R is *base-monotone feasible* if it can be decomposed into pairwise disjoint base-monotone regions with respect to the base-lines. The *k base-line MWRP* is MWRP in which we are given k base-lines, and we find a maximum-weight base-monotone feasible region with respect to the base-lines.

Chun et al. [4] observed that the complement of a maximum-weight base-monotone feasible region represents an object in a picture nicely if the base-lines are located reasonably (see Figs. 1 and 3). They showed that the k base-line MWRP can be solved in polynomial time. In [5], they also studied the *k base-segment MWRP*, in which we are given k segments and find a region decomposable into base-monotone regions with respect to the given base-segments. (We define this problem more precisely in the next section.) They showed some partial results on the complexity of this problem. For other approaches for formulating image segmentation as optimization problems, see e.g. [6,9].

In the setting of the k base-line MWRP, we are given k base-lines. Thus a natural question would be "*What if base-lines are not given?*" In other words, "*How can we divide the pixel grid into subgrids with vertical and horizontal lines?*" We study this problem and show that the problem of optimally locating k base-lines is NP-hard but it can be approximated within factor 2. Next we study the k base-segment MWRP and present sharp contrasts of its computational complexity. Finally, we propose another way for dividing the pixel grid into subgrids, and show that this variant can be solved in polynomial time.

2. Definitions of the problems and our results

In this paper we study three different but well related problems. This section introduces these three problems, briefly explains our results, and then discusses what do the results mean in the context of applications.