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Independent dominating set problem revisited ☆

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A B S T R A C T

An *independent dominating set* of a graph G is a subset D of V such that every vertex not in D is adjacent to at least one vertex of D and no two vertices in D are adjacent. The *independent dominating set (IDS) problem* asks for an independent dominating set with minimum cardinality. First, we show that the independent dominating set problem and the dominating set problem on cubic bipartite graphs are both NP-complete. As an additional result, we give an alternative and more direct proof for the NP-completeness of both the independent dominating set problem and the dominating set problem on at-most-cubic grid graphs. Next, we show that there are fixed-parameter tractable algorithms for the independent dominating set problem and the dominating set problem on at-most-cubic graphs, which run in $O(3.3028^k + n)$ and $O(4.2361^k + n)$ time, respectively. Moreover, we consider the weighted independent dominating set problem on (k, ℓ)-graphs. We show that the problem on $(2, 1)$-graphs is NP-complete. We also show that the problem can be solved in linear time for $(1, 1)$-graphs and in polynomial time for $(1, \ell)$-graphs for constant ℓ, respectively.

1. Introduction

A *dominating set* of a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one vertex of D. An *independent set* of a graph $G = (V, E)$ is a subset I of V such that no two vertices in I are adjacent. An *independent dominating set* of G is a subset of V which is both dominating and independent in G. Equivalently, an independent dominating set is a *maximal* independent set. The *dominating set (DS) problem* asks for a dominating set with minimum cardinality and the *independent dominating set (IDS) problem* asks for an independent dominating set with minimum cardinality. The cardinalities of a minimum dominating set and a minimum independent dominating set are called the *domination number* and the *independent domination number*, respectively. Moreover, the *weighted independent dominating set (WIDS) problem* asks for an independent dominating set D of the given weighted graph such that its weight $w(D) = \sum_{v \in D} w(v)$ is minimum.

A graph is *bipartite* if its vertex set can be partitioned into two independent sets. A graph is *cubic* if every vertex in the graph has degree three. A graph is *at-most-cubic* if the degrees of its vertices are all at most three. A graph is *planar* if it can be embedded in the plane (drawn with points for vertices and curves for edges) without edge-crossings. A graph is *grid* if it is an induced subgraph of a grid. In the former part of this paper, we consider several classes of cubic or at-most-cubic

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graphs. A (k, ℓ)-*graph* is a graph such that its vertex set can be partitioned into at most k independent sets and ℓ cliques. In the latter part of this paper, we turn to consider (k, ℓ)-graphs, whose related work is introduced in Section 5.

First, we describe related work on the IDS problem. Garey and Johnson [16] first showed that the IDS problem for general graphs is NP-complete. Then, Corneil and Perl [10] showed that the IDS problem for bipartite graphs is NP-complete. Later, Clark et al. [9] showed that the IDS problem for grid graphs is NP-complete. In fact, they showed the IDS problem remains NP-complete when restricted to at-most-cubic grid graphs. Zverovich and Zverovich [26] further showed that the IDS problem is NP-complete for at-most-cubic planar bipartite graphs of minimum girth some fixed k, where the *girth* of a graph is the length of a shortest cycle contained in the graph. Moreover, Manlove [22] showed that the IDS problem for cubic planar graphs is NP-complete. Recently, Song et al. [25] showed that the IDS problem for star-convex bipartite graphs is NP-complete, and the IDS problem for triad-convex bipartite graphs can be solved in polynomial time. To the best of our knowledge, the IDS problem for cubic bipartite graphs was open. In this paper, we show that the IDS problem for cubic bipartite graphs is NP-complete. Moreover, via reduction from the rectilinear planar monotone 3SAT problem, we obtain an alternative but more direct proof for the NP-completeness of the IDS problem on at-most-cubic grid graphs.

Furthermore, we mention related work on the approximation results for the IDS problem. Irving [17] showed that the IDS problem for general graphs is not in APX unless $P = NP$. Then, Kann [18] showed that the IDS problem is APX-complete for graphs of maximum degree a constant B. Alimonti and Calamoneri [2] showed that there is an upper bound 2 for approximation ratio of the IDS problem on at-most-cubic graphs. Later, Chlebík and Chlebíková [8] showed that there is a lower bound $\frac{681}{680}$ for approximation ratio of the IDS problem on at-most-cubic graphs.

Next, we describe related work on the DS problem. Garey and Johnson [16] showed that the DS problem for general graphs is NP-complete. Also, in [16], they showed that the DS problems for both at-most-cubic planar graphs and 4-regular planar graphs are NP-complete. Then, Kikuno et al. [19] showed that the DS problem for cubic planar graphs is NP-complete. Clark et al. [9] showed that the DS problem for at-most-cubic grid graphs is NP-complete. Moreover, Zverovich and Zverovich [26] showed that the DS problem is NP-complete for at-most-cubic planar bipartite graphs of minimum girth some fixed k. To the best of our knowledge, the DS problem for cubic bipartite graphs was open. In this paper, we show that the DS problem for cubic bipartite graphs is NP-complete. Moreover, via reduction from the rectilinear planar monotone 3SAT problem, we obtain an alternative but more direct proof for the NP-completeness of the DS problem on at-most-cubic grid graphs.

Moreover, we mention related work on the approximation results for the DS problem. Lund and Yannakakis [21] showed that the DS problem for general graphs is not in APX unless $P = NP$. Baker [4] showed that there is a polynomial-time approximation scheme (PTAS) for the DS problem on planar graphs. Moreover, Papadimitriou and Yannakakis [23] showed that the DS problem for graphs of maximum degree a constant B is APX-complete. Kann [18] showed that the DS problem and the set cover problem are equivalent under L-reduction, which linearly preserves approximability features. Thus according to the result of Duh and Fürer [14] for the set cover problem, an upper bound $\frac{19}{12}$ for approximation ratio of the DS problem on at-most-cubic graphs can be obtained. Later, Chlebík and Chlebíková [8] showed that there is a lower bound $\frac{391}{390}$ for approximation ratio of the DS problem on at-most-cubic graphs.

A problem is *fixed-parameter tractable (FPT)* with respect to parameter k if there exists an algorithmic solution running in $f(k) \cdot n^{O(1)}$ time, where f is a function of the solution size k which is independent of n, and the corresponding algorithm which contributes such a solution is called an *FPT-algorithm*. It is known that both the IDS and DS problems for general graphs are $W[2]$-hard [13]. Thus there are no FPT-algorithms that solve these two problems unless $W[2] = \text{FPT}$.

In the previous paragraphs, we have mentioned that the IDS problem [26] and the DS problem [19] for at-most-cubic graphs are both NP-complete. However, to the best of our knowledge, there was no FPT-algorithm specifically for at-most-cubic graphs. At-most-cubic graphs is closely related to one of its superclasses called 3-degenerate graphs, where a graph G is *d-degenerate* if every subgraph of G has a vertex of degree at most d. From the FPT-algorithms for d-degenerate graphs [3], we first survey on the most efficient known FPT-algorithms for both problems on at-most-cubic graphs in the following paragraphs. Then, we will present more efficient FPT-algorithms for both problems in this paper.

Now we mention related work on the FPT-algorithms for d-degenerate graphs. Alon and Gutner [3] showed that there exists an FPT-algorithm for the DS problem on d-degenerate graphs in $k^{O(dk)} \cdot n$ time. Followed by the above d-degenerate results for $d = 3$, there is an $O(k^{O(3k)} \cdot n)$ algorithm for the DS problem on at-most-cubic graphs. Later, Cygan et al. [11] showed that the IDS problem is fundamentally easier than the DS problem in d-degenerate graphs. It implies that there also exists an $O(k^{O(3k)} \cdot n)$ algorithm for the IDS problem on at-most-cubic graphs.

Next, we need to further introduce a key preprocessing technique called *kernelization* for speeding the running time of FPT-algorithms. It is known that a parameterized problem is in FPT if and only if it has a kernel [13]. By using kernelizations, the input instance can be reduced to a smaller one, that is, a *kernel*, which depends only on the parameter. The first polynomial kernel result on the DS problem is presented by Alber et al. [1]; in fact, their result showed that the DS problem for planar graphs has a linear kernel. As for d-degenerate graphs, Alon and Gutner [3] showed that the DS problem has a kernel of size $k^{O(dk)}$. Later, Philip et al. [24] improved the above kernel size to $O(k^{2(d+1)^2})$. Moreover, a linear kernel of size $4k$ can be easily obtained for at-most-cubic graphs. By combining the FPT-algorithm for 3-degenerate graphs with a kernel of size $4k$, it is clear that there exist $O^*(k^{O(3k)} \cdot 4k) = O^*(k^{O(3k)})$ algorithms for the IDS and DS problems on at-most-cubic graphs. Moreover, the brute-force method will give an $O(2^{4k}k + n) = O(16^k k + n)$ FPT-algorithm for both problems on at-most-cubic graphs. In this paper, we show that the IDS and DS problems on at-most-cubic graphs can be solved more efficiently, say in $O(3.3028^k + n)$ and $O(4.2361^k + n)$ time, respectively.