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On the complexity of the traveling umpire problem ☆

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ABSTRACT

The traveling umpire problem (TUP) consists of determining which games will be handled by each one of several umpire crews during a double round-robin tournament. The objective is to minimize the total distance traveled by the umpires, while respecting constraints that include visiting every team at home, and not seeing a team or venue too often. Even small instances of the TUP are very difficult to solve, and several exact and heuristic approaches for it have been proposed in the literature. To this date, however, no formal proof of the TUP's computational complexity exists. We prove that the decision version of the TUP is $\mathcal{NP}$-complete for certain values of its input parameters.

1. Introduction

The traveling umpire problem (TUP) consists of determining which games will be handled by each one of n umpire crews during a double round-robin tournament with $2n$ teams. The objective is to minimize the total distance traveled by the umpires, while respecting constraints that include visiting every team at home, and not seeing a team or venue too often throughout the season. The TUP was created as an abstraction of the real-life umpire scheduling problem faced by Major League Baseball in an attempt to isolate the few features that make the problem difficult to solve (see [1]). Since it was first introduced, several papers have proposed exact and heuristic approaches to tackle the TUP, such as [1–6]. Despite the steady progress in solving progressively larger instances of the problem, empirical evidence shows that the TUP is still a very difficult problem to solve. According to the official TUP benchmark set [7], no instances with more than 10 teams have known optimal solutions.

On the theoretical side, however, the TUP has attracted far less attention. To this date, no formal proof of the TUP's computational complexity exists, and this is the focus of our paper. We are concerned with the decision version of the TUP, as defined below.

Definition 1. Given a double round-robin tournament T with $2n$ teams, the distance d_{ij} between the home venues of any two teams i and j, two non-negative integers $d_1 \le n-1$ and $d_2 \le \lfloor n/2 \rfloor - 1$, and a non-negative number ℓ, the decision version of the TUP consists of determining whether or not there exists an assignment of n umpire crews (umpires, for short) to the games of T that satisfies all of the following conditions:

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$U_{8,0}$

Rounds						
0	1	2	3	4	5	6
(0, 7)	(1, 7)	(2, 7)	(3, 7)	(4, 7)	(5, 7)	(6, 7)
(1, 6)	(2, 0)	(3, 1)	(4, 2)	(5, 3)	(6, 4)	(0, 5)
(2, 5)	(3, 6)	(4, 0)	(5, 1)	(6, 2)	(0, 3)	(1, 4)
(3, 4)	(4, 5)	(5, 6)	(6, 0)	(0, 1)	(1, 2)	(2, 3)

$U_{8,8}$

Rounds						
0	1	2	3	4	5	6
(8, 15)	(9, 15)	(10, 15)	(11, 15)	(12, 15)	(13, 15)	(14, 15)
(9, 14)	(10, 8)	(11, 9)	(12, 10)	(13, 11)	(14, 12)	(8, 13)
(10, 13)	(11, 14)	(12, 8)	(13, 9)	(14, 10)	(8, 11)	(9, 12)
(11, 12)	(12, 13)	(13, 14)	(14, 8)	(8, 9)	(9, 10)	(10, 11)

$U_{8,16}$

Rounds						
0	1	2	3	4	5	6
(16, 23)	(17, 23)	(18, 23)	(19, 23)	(20, 23)	(21, 23)	(22, 23)
(17, 22)	(18, 16)	(19, 17)	(20, 18)	(21, 19)	(22, 20)	(16, 21)
(18, 21)	(19, 22)	(20, 16)	(21, 17)	(22, 18)	(16, 19)	(17, 20)
(19, 20)	(20, 21)	(21, 22)	(22, 16)	(16, 17)	(17, 18)	(18, 19)

$U_{8,24}$

Rounds						
0	1	2	3	4	5	6
(24, 31)	(25, 31)	(26, 31)	(27, 31)	(28, 31)	(29, 31)	(30, 31)
(25, 30)	(26, 24)	(27, 25)	(28, 26)	(29, 27)	(30, 28)	(24, 29)
(26, 29)	(27, 30)	(28, 24)	(29, 25)	(30, 26)	(24, 27)	(25, 28)
(27, 28)	(28, 29)	(29, 30)	(30, 24)	(24, 25)	(25, 26)	(26, 27)

Fig. 1. Tournaments $U_{8,0}$, $U_{8,8}$, $U_{8,16}$, and $U_{8,24}$.

(i) In every round of T, each umpire is assigned to exactly one game, and each game is assigned to exactly one umpire;
(ii) Each umpire visits the home venue of every team at least once;
(iii) No umpire visits a venue more than once in any sequence of $n - d_1$ consecutive rounds;
(iv) No umpire sees a team more than once in any sequence of $\lfloor n/2 \rfloor - d_2$ consecutive rounds;
(v) The total distance traveled by the n umpires during T is less than or equal to ℓ.

Our main contribution is to prove that the decision version of the TUP is an $\mathcal{NP}$-complete problem when $d_1 \le n/2$ and $d_2 = \lfloor n/2 \rfloor - 1$.

The remainder of this paper is organized as follows. Section 2 introduces some notation used throughout the paper and establishes a few preliminary results. Section 3 presents an $\mathcal{NP}$-complete problem that can be reduced to the TUP, followed by the proof of our main result. Finally, we conclude the paper and propose future research directions in Section 4.

2. Notation and preliminary results

In this section we introduce some notation that will be used in our main result and prove a number of auxiliary results.

Let T be a tournament with $2n$ teams and m rounds. Then, T can be defined as a sequence of sets of ordered pairs by writing $T = S_0, S_1, \ldots, S_{m-1}$, where S_s contains the games that take place in the $(s+1)$-th round.[1] We assume that the first team in each ordered pair is the home team. Let $C = \{(i_0, j_0), (i_1, j_1), \ldots, (i_{v-1}, j_{v-1})\}$ be a set with v ordered pairs. We denote by $\overline{C}$ the set obtained from C by reversing the order of the elements in each ordered pair in C. Therefore, $\overline{C} = \{(j_0, i_0), (j_1, i_1), \ldots, (j_{v-1}, i_{v-1})\}$. Using this notation, the reversal of the home venues of T can be denoted by $\overline{T} = \overline{S}_0, \overline{S}_1, \ldots, \overline{S}_{m-1}$. In other words, for every pair of teams i and j, if i plays at home against j in round s of T, then j plays at home against i in round s of $\overline{T}$.

A single (double) round-robin tournament is a tournament in which each team plays against each other team exactly once (twice: once at each team's home venue). Eqs. (1)–(3) define a constructive way of creating a single round-robin tournament $U_{a,b}$ with an even number of teams $a \ge 2$, $a-1$ rounds, and team IDs ranging from b to $b+a-1$:

$$U_{a,b} = U_{a,b}[0, a-2], \tag{1}$$

$$U_{a,b}[s_1, s_2] = Q_{a,b}[s_1], Q_{a,b}[s_1+1], \ldots, Q_{a,b}[s_2], \quad \forall 0 \le s_1 \le s_2 \le a-2, \tag{2}$$

$$\begin{aligned} Q_{a,b}[s] = \Big\{ & \big(b + (s \bmod (a-1)), b + a - 1\big), \\ & \big(b + ((s+1) \bmod (a-1)), b + ((s+a-2) \bmod (a-1))\big), \\ & \big(b + ((s+2) \bmod (a-1)), b + ((s+a-3) \bmod (a-1))\big), \\ & \vdots \\ & \big(b + ((s+a/2-1) \bmod (a-1)), b + ((s+a-a/2) \bmod (a-1))\big)\Big\}, \quad \forall 0 \le s \le a-2. \end{aligned} \tag{3}$$

This algebraic definition results in a method equivalent to the well-known circle/polygon method, also known in literature as Kirkman's method, which was first introduced in [8]. Fig. 1 illustrates four 8-team $U_{a,b}$ tournaments, and Lemma 1 asserts the correctness of (1)–(3).

[1] Although we use round indices starting at zero, we avoid referring to round 0 as the zero-th round. Hence, S_0 is the first round, S_1 is the second round, and so on. The same applies to other ordinal indices throughout the paper.