Theoretical Computer Science 562 (2015) 146–164

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

One-to-one disjoint path covers on alternating group graphs

Lantao You [a], Jianxi Fan [a,*], Yuejuan Han [b], Xiaohua Jia [c]

[a] School of Computer Science and Technology, Soochow University, Suzhou 215006, China
[b] Center of Information Development and Management, Soochow University, Suzhou 215006, China
[c] Department of Computer Science, City University of Hong Kong, Hong Kong

ARTICLE INFO

Article history:
Received 24 April 2014
Received in revised form 11 August 2014
Accepted 17 September 2014
Available online 28 September 2014
Communicated by G. Ausiello

Keywords:
Alternating group graph
One-to-one disjoint path covers
Interconnection network
Parallel computing

ABSTRACT

The alternating group graph, denoted by AG_n, is one of the popular interconnection networks, which has many attractive properties. In this paper, we prove that for any two distinct nodes μ and ν, there exist m node-disjoint paths for any integer $n \geq 3$ with $1 \leq m \leq 2n-4$ whose union covers all the nodes of AG_n. For any node of AG_n has exactly $2n-4$ neighbors, $2n-4$ is the maximum number of node-disjoint paths can be constructed in AG_n.

1. Introduction

An interconnection network can be represented by a simple graph $G = (V, E)$, where V represents the node set and E represents the edge set. So far, interconnection networks have been widely studied [4,5,8–10,16–19,23,24].

A disjoint path cover (DPC for short) of a graph is the node-disjoint path(s) between two distinct nodes, whose union covers all the nodes of the graph [7]. Finding node-disjoint paths is one of the important research topics in various interconnection networks. The node-disjoint paths can be used to speed up data transfer and avoid communication congestion by providing parallel communication paths. Additional benefits of adopting such a node-disjoint routing scheme are the enhanced robustness to node failures, and the enhanced capability of load balancing [12]. Another well-known application of multiple disjoint path covers is software testing [11].

An n-dimensional alternating group graph, denoted by AG_n, was proposed by Jwo et al. [6]. The alternating group graphs have many attractive properties such as node-transitivity, edge-transitivity, maximal connectivity and small diameter, and have been used as the underlying topology for many practical multicomputers [13]. Jwo et al. [6] proved that AG_n is pancyclic and hamiltonian-connected for any integer $n \geq 3$. Chang et al. [2,3] proved that AG_n is $(n-2)$-node fault-tolerant pancyclic, $(n-3)$-node fault-tolerant node-pancyclic and $(n-4)$-node fault-tolerant edge-4-pancyclic for any integer $n \geq 4$. Zhou et al. investigated the properties of AG_n about fault tolerance [22], conditional diagnosability [20] and node-disjoint paths [21]. Su et al. showed that for any integer $n \geq 3$, AG_n contains $2n-4$ mutually independent hamiltonian cycles [13]. Szepietowski showed that the optimal upper bound on fault tolerance of edge 5-pancyclicity is equal to $n-3$ and it jumps up to $2n-7$ for edge 6-pancyclicity [14]. So far there is no work reported about the one-to-one disjoint path covers properties of AG_n.

* Corresponding author.
E-mail addresses: yoult@siso.edu.cn (L. You), jxfan@suda.edu.cn (J. Fan).

http://dx.doi.org/10.1016/j.tcs.2014.09.041

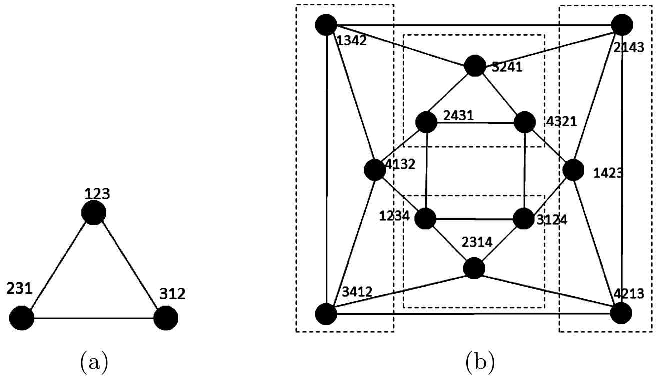

Fig. 1. (a) The alternating group graph AG_3; (b) the alternating group graph AG_4.

In this paper, we study the one-to-one disjoint path covers properties for AG_n, and we prove that given any two distinct nodes μ, ν of AG_n, there exist(s) m node-disjoint path(s) between μ and ν whose union covers all nodes of AG_n for any integer $n \geq 3$ with $1 \leq m \leq 2n-4$. For any node of AG_n has exactly $2n-4$ neighbors, $2n-4$ is the maximum number of node-disjoint paths can be constructed in AG_n. The rest of this paper is organized as follows. In Section 2, we give the notation used in this paper and provide the formal definition of the alternating group graph, and then present some basic properties of it. In Section 3, we give the proofs of our main results. We summarize the paper in Section 4 and give Appendix A.

2. Preliminaries

In this paper, we follow [1] for the graph definition and notation. For a given interconnection network $G = (V, E)$, an adjacent node of a node μ in a graph is a node that is joined to μ by an edge. A path P between two nodes μ_1 and μ_k is represented by $P = \langle \mu_1, \mu_2, ..., \mu_k \rangle$, where each pair of consecutive nodes are joined by an edge. A path that contains every node of G exactly once is called a hamiltonian path. Similarly, a hamiltonian cycle of G is a cycle that contains every node of G exactly once. We call G a hamiltonian graph if G has a hamiltonian cycle.

Suppose that μ and ν are any two nodes in G, a set of m paths between μ and ν is an m-disjoint path cover in G if the m paths do not contain the same nodes except μ and ν, and their union covers all nodes of G. An m-disjoint path cover for simplicity is abbreviated as an m-DPC. A graph G is one-to-one m-disjoint path coverable (m-DPC-able for short) if there is an m-DPC between any two nodes of G.

Let $\langle n \rangle$ be the set $\{1, 2, ..., n\}$ where $n \geq 3$, and let $p = p_1 p_2 ... p_n$ where $p_i \in \langle n \rangle$ and $p_i \neq p_j$ for $i \neq j$. Here p is a permutation of elements in $\langle n \rangle$, where p_i denotes the element at the position i for $1 \leq i \leq n$. A pair of elements p_i and p_j in p is said to be an inversion of p if $p_i < p_j$ whenever $i > j$. A permutation is an even permutation if it has an even number of inversions. Take $p = 13425$ for example, for the pairs $(3, 2)$ and $(4, 2)$ are two inversions, p is an even permutation. Let A_n denote the set of all even permutations over $\langle n \rangle$, for any integer $n \geq 3$ and $3 \leq i \leq n$, we define two operations, g_i^+ and g_i^-, on A_n by setting pg_i^+ (resp. pg_i^-) to be the permutation obtained from p by rotating the symbols in positions 1, 2, and i from left to right (resp. from right to left). For example, if $p = 13425$, then $pg_4^+ = 21435$ and $pg_4^- = 32415$. Each node of AG_n is denoted by an n-bit label $p_1 p_2 p_3 ... p_n$, we will use this label to represent the node in the following paper.

Definition 1. (See [6].) For any integer $n \geq 3$, an n-dimensional alternating group graph, AG_n, is defined as follows:

(1) AG_3 is a graph consisting of three nodes labelled with 123, 231, 312, respectively, joined by three edges $(123, 312)$, $(123, 231)$ and $(231, 312)$.

(2) For any integer $n \geq 4$, AG_n is built recursively by using n copies of AG_{n-1}. Let AG_n^i denote the subgraph of AG_n induced by the nodes with the rightmost bit being i, thus, AG_n is partitioned into n copies $AG_n^1, AG_n^2, ..., AG_n^n$ of AG_{n-1}. The n-dimensional alternating group graph AG_n is a graph consisting of the node set $V(AG_n)$ denoted by A_n and two nodes $p, q \in A_n$ are adjacent if and only if $q = pg_i^+$ (p_i^+ for short) or $q = pg_i^-$ (p_i^- for short) for some $i = 3, 4, ..., n$. Fig. 1(a) depicts AG_3 and Fig. 1(b) depicts AG_4, respectively.

AG_n is node symmetric and edge symmetric. It is obviously that AG_n is a regular graph with $n!/2$ nodes, $(n-2)n!/2$ edges and node degree $2n-4$. For any integer $n \geq 3$, edges joining nodes in the same subgraph AG_n^i with any integer $1 \leq i \leq n$ are called internal edges and edges joining nodes in different subgraphs are called external edges. Clearly, each node of AG_n^i with $1 \leq i \leq n$ is joined to exactly two external edges and $2n-6$ internal edges.

The following properties further describe alternating group graphs [2]: