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Primal–dual approximation algorithm for the two-level facility location problem via a dual quasi-greedy approach

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ABSTRACT

The main contribution of this work is to propose a primal–dual combinatorial $3(1+\varepsilon)$-approximation algorithm for the *two-level facility location problem* (2-LFLP) by exploring the approximation oracle concept. This result improves the previous primal–dual 6-approximation algorithm for the multilevel facility location problem, and also matches the previous primal–dual approximation ratio for the single-level facility location problem. One of the major merits of primal–dual type algorithms is their easy adaption to other variants of the facility location problems. As a demonstration, our primal–dual approximation algorithm can be easily adapted to several variants of the 2-LFLP, including models with stochastic scenario, dynamically arrived demands, and linear facility cost.

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1. Introduction

Facility location problems have been extensively studied in the operation research and computer science communities due to their vast applications. In the classical *single-level facility location problem* (1-LFLP), there are a set of facilities and a set of clients. Our objective is to open a subset of facilities and connect every client to an open facility such that the total cost of opening facilities and connecting clients is minimized. In the *k-level facility location problem* (k-LFLP), the facilities form a hierarchy of k levels, and a facility *path* consists of exactly one facility from each level in the hierarchical order from the lowest to the highest level. The objective is to open k subsets of facilities in each level and connect every client to an open path (a facility path consisting of only open facilities) such that the total cost of opening facilities and connecting clients is minimized. Throughout this paper, we assume that the connection costs are metric, that is, nonnegative, symmetric, and satisfying the triangle inequality.

It is well-known that the k-LFLP for any $k \geq 1$ is NP-hard and hence approximation algorithms have been one of the important methods in solving this problem. However, it was known that one cannot have an approximation ratio smaller than 1.463 for the 1-LFLP unless $P = NP$ [13,23], smaller than 1.539 for the 2-LFLP, and smaller than 1.61 for the general k-LFLP unless $NP \subseteq DTIME(n^{O(\log\log n)})$ [16].

On the positive side, there exist many constant approximation algorithms in the literature. For the 1-LFLP, the first constant 3.16-approximation algorithm is due to Shmoys et al. [21], followed by many improvements over the years [4,6,9,14,15,19,24], with the currently best approximation ratio 1.488 by Li [17]. For the 2-LFLP, Zhang [29] gives a dual-fitting based

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(but non-combinatorial) approximation algorithm with the currently best ratio $1.77(1+\varepsilon)^2$ by using a combination of quasi-greedy and LP-rounding techniques, while the best combinatorial approximation ratio 2.4211 is due to Ageev et al. [2]. The best approximation for the k-LFLP, for any constant $k \geq 3$, is given by Byrka and Rybicki [5]. For the general k-LFLP, Ageev et al. [2] give a 3.27-approximation algorithm which is the currently best combinatorial approximation algorithm, while both Aardal et al. [1] and Gabor and van Ommeren [11] give the currently best non-combinatorial approximation ratio of 3 based on LP-rounding techniques, where the former is based on an LP formulation with an exponential number of variables, and the latter is based on a compact LP formulation with polynomial number of decision variables. Moreover, Bumb and Kern [3] give a primal–dual combinatorial 6-approximation algorithm which generalizes the primal–dual approach of Jain and Vazirani [15].

1.1. Problem description

The focus of this work is on the 2-LFLP, which can be formally described as follows. Given two level sets of facilities $\mathcal{F}_1$ and $\mathcal{F}_2$ and a set of clients $\mathcal{D}$, there are facility open costs f_i and f_k for opening facilities $i \in \mathcal{F}_2$ and $k \in \mathcal{F}_1$, respectively; and there are connection costs c_{ik} between $i \in \mathcal{F}_2$ and $k \in \mathcal{F}_1$, and c_{kj} between $k \in \mathcal{F}_1$ and $j \in \mathcal{D}$, respectively. We need to open some facilities in $\mathcal{F}_1$ and $\mathcal{F}_2$ to compose some open paths, and connect each client to an open path such that the facility (opening) cost and connection cost is minimized. For convenience, we introduce the following notations:

$$c_{ikj} := c_{ik} + c_{kj}, \quad \forall i \in \mathcal{F}_2,\ k \in \mathcal{F}_1,\ j \in \mathcal{D};$$
$$C_{iSD} := f_i + \sum_{k\in S} f_k + \sum_{j\in D} \min_{k\in S} c_{ikj}, \quad \forall i \in \mathcal{F}_2,\ S \subseteq \mathcal{F}_1,\ D \subseteq \mathcal{D}.$$

Since there always exists an optimal solution that is a forest for the 2-LFLP [10], we can formulate the 2-LFLP as the following binary integer linear program.

$$\begin{aligned} \min \quad & \sum_{i\in\mathcal{F}_2} \sum_{S\subseteq\mathcal{F}_1} \sum_{D\subseteq\mathcal{D}} C_{iSD} X_{iSD} \\ \text{s.t.} \quad & \sum_{i\in\mathcal{F}_2} \sum_{S\subseteq\mathcal{F}_1} \sum_{D\subseteq\mathcal{D}: j\in D} X_{iSD} \geq 1, \quad \forall j \in \mathcal{D}, \\ & X_{iSD} \in \{0,1\}, \quad \forall i \in \mathcal{F}_2,\ S \subseteq \mathcal{F}_1,\ D \subseteq \mathcal{D}. \end{aligned} \tag{1.1}$$

By relaxing the integrality constraints, we get the linear program relaxation, whose dual program is as follows.

$$\begin{aligned} \max \quad & \sum_{j\in\mathcal{D}} \alpha_j \\ \text{s.t.} \quad & \sum_{j\in D} \alpha_j \leq C_{iSD}, \quad \forall i \in \mathcal{F}_2,\ S \subseteq \mathcal{F}_1,\ D \subseteq \mathcal{D}, \\ & \alpha_j \geq 0, \quad \forall j \in \mathcal{D}. \end{aligned} \tag{1.2}$$

1.2. Our results and techniques

One key component in the standard primal–dual algorithm [15] is to identify the time at which one of the dual constraints becomes tight whenever the dual variables are updated. This identification problem, a.k.a *separation* problem in the literature [12], for the dual LP (1.2) for the 1-LFLP is easily solvable in polynomial-time, in contrast to the situation for the 2-LFLP, where we are facing the Max-1-LFLP, an NP-hard problem. One possible way to overcome this difficulty is to approximately solve the separation problem, e.g., using the technique in [29]. Unfortunately a straightforward combination of the primal–dual algorithm [15] with the quasi-greedy approach [29] is not possible. The main reason is that the latter approach in [29] is only applicable in the primal-greedy (or the dual-fitting) setting [14] where dual feasibility is not required. On the other hand what we really need here is a 'dual' version of the quasi-greedy approach (details in Section 3.1), in order to be integrated with dual-greedy type of approaches such as the primal–dual method [15], where dual feasibility must be respected throughout the algorithm.

Using an analog in the classical set covering problem, the aforementioned *primal-* and *dual-greedy* approaches lead to approximation ratios $\ln(n)$ and f, respectively, where n is the size of the ground set and f is the maximum frequency of elements. For details on the set covering problem, the readers are referred to [26]: Chapters 2 and 15, in which the more popular terms *greedy* algorithm and *primal–dual* algorithm are used, respectively.

The main result of this work is to obtain a primal–dual combinatorial $3(1+\varepsilon)$-approximation algorithm, improving the previous primal–dual 6-approximation algorithm for the k-LFLP of Bumb and Kern [3], and also matching the previous primal–dual approximation ratio for the 1-LFLP of Jain and Vazirani [15]. Furthermore, by exploring the so-called Add operation in [29], we can improve the ratio from $3(1+\varepsilon)$ to $2.172(1+\varepsilon)^2$.

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