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Randomized algorithms for online knapsack problems ☆

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ABSTRACT

In this paper, we study online knapsack problems. The input is a sequence of items $e_1, e_2, \dots, e_n$, each of which has a size and a value. Given the ith item e_i, we either put e_i into the knapsack or reject it. In the removable setting, when e_i is put into the knapsack, some items in the knapsack are removed with no cost if the sum of the size of e_i and the total size in the current knapsack exceeds the capacity of the knapsack. Our goal is to maximize the profit, i.e., the sum of the values of items in the last knapsack.

We present a simple randomized 2-competitive algorithm for the unweighted non-removable case and show that it is the best possible, where knapsack problem is called unweighted if the value of each item is equal to its size.

For the removable case, we propose a randomized 2-competitive algorithm despite there is no constant competitive deterministic algorithm. We also provide a lower bound $1 + 1/e \approx 1.368$ for the competitive ratio. For the unweighted removable case, we propose a 10/7-competitive algorithm and provide a lower bound 1.25 for the competitive ratio.

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1. Introduction

The knapsack problem is one of the most fundamental problems in combinatorial optimization and has a lot of applications in the real world [14]. The knapsack problem is that: given a set of items e_i with values $v(e_i)$ and sizes $s(e_i)$, we are asked to maximize the total value of selected items in the knapsack satisfying the capacity constraint. Throughout this paper, we assume that the capacity of knapsack is 1.

In this paper, we study the online version of the knapsack problem. Here, "online" means that i) the information of the input (i.e., the items) is given gradually, i.e., after a decision is made on the current item, the next item is given; ii) the decisions we have made are irrevocable, i.e., once a decision has been made, it cannot be changed. Given the ith item e_i, which has a value $v(e_i)$ and a size $s(e_i)$, we either accept e_i (i.e., put e_i into the knapsack) or reject it. In the removable setting, when e_i is put into the knapsack, some items in the knapsack are removed with no cost if the sum of the sizes of e_i and the total size in the current knapsack exceeds 1 (i.e., the capacity of the knapsack). Our goal is to maximize the profit, i.e., the sum of the values of items in the last knapsack.

Related works It is well known that offline knapsack problem is NP-hard but admits an FPTAS. Ito et al. [11] presented a constant-time randomized approximation algorithm by using weighted sampling.

☆ An extended abstract of this paper appears in FAW/AAIM 2013 [9].

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Table 1
The current status on competitive ratios for online knapsack problems, where our results are written in bold letters.

		Unweighted		General	
		Lower bound	Upper bound	Lower bound	Upper bound
non-removable	deterministic	∞ [12]		∞ [16]	
	randomized	**2**		∞ [18]	
removable	deterministic	$\frac{1+\sqrt{5}}{2}$ [12]		∞ [13]	
	randomized	**5/4**	**10/7**	$\mathbf{1+\frac{1}{e}}$	**2**

An online knapsack problem was first studied on average case analysis by Marchetti-Spaccamela and Vercellis [16]. They proposed a linear-time algorithm with $O(\log^{3/2} n)$ expected competitive difference, under the condition that the capacity of the knapsack grows proportionally to the number of items n. Lueker [15] improved the expected competitive difference to $O(\log n)$ under a fairly general condition on the distribution.

On the worst case analysis, Marchetti-Spaccamela and Vercellis [16] showed that general online knapsack problem has no constant (deterministic) competitive ratio. Buchbinder and Naor [4] presented an $O(\log(U/L))$-competitive algorithm based on a general online primal-dual framework when the density of every element is in a known range $[L, U]$, and each size is assumed to be much smaller than the capacity of the knapsack. They also showed an $\Omega(\log(U/L))$ lower bound on the competitive ratio for the case. Zhou et al. [18] showed $\Omega(\log(U/L))$ is also lower bound for the randomized case, which implies that general online knapsack problem has no constant randomized competitive ratio.

Iwama and Taketomi [12] studied the *removable* online knapsack problem. They obtained a $(1+\sqrt{5})/2 \approx 1.618$-competitive algorithm for the unweighted online knapsack, where knapsack problem is called *unweighted* if the value of each item is equal to its size, and showed that this is the best possible by providing a lower bound $(1+\sqrt{5})/2$ for the case. We remark that the problem has unbounded competitive ratio, if at least one of the removal and unweighted conditions is not satisfied [12,13]. For the randomized competitive ratio of the general removable online knapsack problem, Babaioff et al. [2] showed a lower bound 5/4.

Removable online knapsack problem with cancellation cost is studied in [1,2,8]. When the cancellation cost is proportional, i.e., it is f times the total value of removed items, Babaioff et al. [1,2] showed that if each item has size at most γ, where $0 < \gamma < 1/2$, then the competitive ratio is at most $1 + 2f + 2\sqrt{f(1+f)}$ with respect to the optimal solution for the knapsack problem with capacity $(1 - 2\gamma)$. They also proposed a randomized $3(1 + 2f + 2\sqrt{f(1+f)})$-competitive algorithm for this problem. Han et al. [8] showed that unweighted version of this problem is $\max\{2, \frac{1+f+\sqrt{f^2+2f+5}}{2}\}$-competitive.

For the other models such as knapsack secretary problem, stochastic knapsack problem and minimization knapsack problem, refer to papers in [3,6–8,10].

1.1. Our results

In this paper, we study the worst case analysis of randomized algorithms for online knapsack problem against an oblivious adversary.

We first provide a randomized 2-competitive algorithm for the unweighted non-removable online knapsack problem, and show that it is the best possible.

For the unweighted removable case, we propose a randomized 10/7-competitive algorithm. Our algorithm divides all the items into three groups, *small*, *medium* and *large*. If a large item comes, our algorithm chooses it and cancels all the items in the knapsack. Otherwise the algorithm first handles medium items, then apply a greedy algorithm for the small items. For medium items, it randomly selects the one among two deterministic subroutines. We also show that there exists no randomized online algorithm with competitive ratio less than 5/4 for the unweighted removable case.

For the general removable case, we present a simple randomized 2-competitive algorithm, which is an extension of famous 2-approximation greedy algorithm for offline knapsack problem. As a lower bound, we show that there exists no randomized online algorithm with competitive ratio less than $1 + 1/e$ for the general removable online knapsack problem.

We mention that Cygan and Jeż [5] have independently and concurrently studied the online knapsack problem. The lower bound of the competitive ratios 5/4 and $1 + 1/e$ for the unweighted and general removable cases are also shown in the paper.

We summarize the current status on competitive ratios for the online knapsack problem in Table 1, where our results are written in bold letters.

The rest of paper is organized as follows. In Section 2, we provide competitive ratio for the unweighted non-removable cases. In Sections 3 and 4, we consider the unweighted and general removable cases, respectively.

2. Unweighted non-removable online knapsack problem

In this section we study the non-removable version of the unweighted online knapsack problem. We show that the problem is randomized 2-competitive.

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