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Inapproximability of dominating set on power law graphs

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ABSTRACT

We prove the first logarithmic lower bounds for the approximability of the Minimum Dominating Set problem for the case of connected (α, β)-power law graphs for α being a size parameter and β the power law exponent. We give also a best up to now upper approximation bound for this problem in the case of the parameters $\beta > 2$. We develop also a new functional method for proving lower approximation bounds and display a sharp approximation phase transition area between approximability and inapproximability of the underlying problems. Our results depend on a method which could be also of independent interest.

1. Introduction

The Minimum Dominating Set problem (Min-DS) asks for a minimum size set of vertices D for a given graph G such that each vertex in G is either contained in D or adjacent to some vertex in D. The Min-DS problem has asymptotically the same approximation upper and lower bounds as the Set Cover problem. It can be approximated within $(1 - o(1))\ln(n)$ by a greedy algorithm and, unless $\mathsf{NP} \subseteq \mathsf{DTIME}(n^{O(\log\log n)})$, there is no $(1 - \varepsilon)\ln(n)$-approximation algorithm for Min-DS for any $\varepsilon > 0$ [8]. Furthermore, Raz and Safra established an approximation lower bound of $c \cdot \ln(n)$ for some constant c under the weaker assumption that $\mathsf{P} \neq \mathsf{NP}$ [20].

In this paper we give new approximation upper and lower bounds for Min-DS on power law graphs. G is called a *power law graph* if the number of nodes of degree i is proportional to $i^{-\beta}$, for some $\beta > 0$. The parameter β is called the *power law exponent* and determines the log–log growth rate of G. The Min-DS problem on power law graphs was originally introduced in the context of the sensor placement problems in massive social networks (cf. [7]).

Power law graphs (PLG) have been used in modeling and analyzing the real-world networks like the graphs of the Internet and the World Wide Web (WWW), peer-to-peer networks, mobile call networks, protein–protein interaction networks, gene regulatory networks, food webs and various social networks. Typically, the power law exponent of these real-world networks lies within the range $2 < \beta < 3$ (e.g. $\beta = 2.38$ for the WWW [5], $\beta = 2.4$ for protein–protein interaction networks [13]). There also exist examples of real-world networks with a power law exponent $\beta \leq 2$ or $\beta \geq 3$, e.g. for distributional food webs ($\beta = 1.05$, [18]), statistical investigations of book sales in the US ($\beta = 3.51$, [12,19]) and human contact networks ($\beta = 3.4$, [17]).

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Table 1
Summary of the main results: approximation lower bounds and approximation upper bounds for MIN-DS on (α,β)-PLG for certain ranges of the parameter β. The precise choice of the parameter d is described in Theorem 4.

Power law exponent	Approx. lower bound
$0<\beta<1$	$\Omega(\ln(n)-\ln(\frac{1}{1-\beta}))$
$\beta=1$	$\Omega(\ln(n))$
$1<\beta<2$	$\Omega(\ln(n)-\ln(\zeta(\beta)))$
$\beta=2$	$\Omega(\ln(n)-\ln(\zeta(\beta)))$
$\beta=2+\frac{1}{f(n)}$, $f(n)=\omega(\log n)$	$\Omega(\ln(n)-\ln(\zeta(\beta)))$

Power law exponent	Approx. upper bound
$\beta=2+\frac{1}{f(n)}$, $f(n)=o(\log n)$	APX
$2<\beta\le 2.729$	$\frac{\zeta(\beta)-1}{\zeta(\beta)-\sum_{j=1}^{d-1} j^{-\beta}}$
$\beta>2.729$	$\frac{\zeta(\beta-1)-2\zeta(\beta)}{\zeta(\beta-1)-2}$

A number of different random graph models were proposed in order to capture the topological properties of real-world networks and to analyze these graphs on the basis of a so called *null-model* (see [1–4,6,15,16]). On this basis, two different types of models have been introduced. The *evolving models* define a random process where one node at a time is added and connected to the existing graph in a random fashion—and thus are aiming to describe how power laws arise. The *static models* start from a given power law degree sequence as an input and then perform a random selection from the space of graphs with this degree sequence. The most prominent examples of the two types are the *preferential attachment model* described by Barabási [3], and the *ACL model* introduced by Aiello, Chung, and Lu [1,2].

In this paper, we consider the power law model (α,β)-PLG due to [1] (also called the ACL model). A (multi-)graph G with maximum degree Δ is called an (α,β)-PLG with size parameter α and a power law exponent β, if for each $i\le\Delta=\lfloor e^{\alpha/\beta}\rfloor$, the number of nodes of degree i is equal to $\lfloor e^{\alpha}/i^{\beta}\rfloor$.

2. Previous results

Ferrante, Pandurangan, and Park [9] have shown the NP-hardness of MIN-DS on simple disconnected (α,β)-PLG for $\beta>0$. In [21] it was shown that MIN-DS on (α,β)-PLG is in APX for $\beta>2$. Furthermore, for $\beta>1$, APX-hardness was shown and explicit constant approximation lower bounds were given, namely $1+\frac{1}{390(2\zeta(\beta)3^{\beta}-1)}$ on (α,β)-PLG multigraphs and $1+\frac{1}{3120\zeta(\beta)3^{\beta}}$ on simple (α,β)-PLG.

Eubank et al. [7] studied a relaxed version of MIN-DS: In the $(1-\varepsilon)$-MIN-DS problem the requirement is to dominate at least a $(1-\varepsilon)$-fraction of the vertices. They show that for every $\varepsilon>0$, the $(1-\varepsilon)$-MIN-DS problem on bipartite random PLG admits a PTAS.

3. Our results

In this paper, we give the first logarithmic lower approximation bounds for MIN-DS on (α,β)-PLG for the case $\beta\le 2$. The best up to now approximation lower bound was a constant bound [21]. We show that in this case, unless NP $\subseteq$ DTIME$(n^{O(\log\log n)})$, MIN-DS on connected (α,β)-PLG cannot be approximated within an approximation ratio $\Omega(\ln(n))$. Thus our lower approximation bound is almost tight. We also give improved approximation upper bounds for the case $\beta>2$ and show that in this case, MIN-DS on (α,β)-PLG can be approximated within some constant approximation ratio R_{β} which converges to 1 as $\beta\to\infty$.

Then we take a very precise look at the phase transition point at $\beta=2$. We consider a case when $\beta=2+1/f(n)$ is a function of the size n of the graph. Here, n denotes the number of vertices of the PLG, and f is a monotone increasing unbounded function. This is an extension of the (α,β)-PLG model in [1], for which β was always a fixed constant. Surprisingly, we obtain a very sharp phase transition result, between approximability and inapproximability areas depending on the order of magnitude of the function f. We show that when $f(n)=o(\log n)$, MIN-DS on $(\alpha,2+1/f(n))$-PLG is still in APX. On the other hand, we give a logarithmic approximation lower bound for the case when $f(n)=\omega(\log n)$.

Our approximation lower bounds are based on a direct approximate reduction from the SET COVER problem to the MIN-DS problem combined with an embedding of the resulting graph instances into (α,β)-PLG. Our constructions rely on precise estimates of sizes of node intervals in (α,β)-PLG and on the available node degree inside these intervals. Table 1 summarizes our main results in lower and upper approximation bounds for MIN-DS on (α,β)-PLG.

4. Organization of the paper

In Section 5, we are giving an outline of the proof methods and the simulating constructions on which our reductions are based. In Section 6, we use the original reduction of Feige [8] from 5OCC-MAX-E3-SAT (5 OCCURRENCE MAXIMUM E3-SAT)