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Hamiltonicity of the basic WK-recursive pyramid with and without faulty nodes

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ABSTRACT

In 1993, Fernandes and Kanevsky proposed an important structure for interconnection networks, WKR Pyramid Networks ($\mathrm{WKP}_{(d,t,L)}$, for short). These are constructed by taking difference size WK-recursive network ($\mathrm{WK}_{(d,tn)}$, for short) as difference layers. That study discussed the orders, sizes and connectivity of $\mathrm{WKP}_{(d,t,L)}$ for any integers $d \geq 1, t \geq 1$ and $L \geq 1$. The basic WK-recursive pyramid, denoted by $\mathrm{WKP}_{(d,L)}$, is a basic version of $\mathrm{WKP}_{(d,t,L)}$ such that $t = 1$. In $\mathrm{WKP}_{(d,L)}$, each vertex has exactly d children and the nth layer is isomorphic to a $\mathrm{WK}_{(d,n)}$. In this paper, we show that $\mathrm{WKP}_{(d,L)}$ is Hamiltonian-connected for any two integers, $d \geq 3$ and $L \geq 1$, and it is also $(d-2)$-node Hamiltonian for any two integers, $d \geq 2$ and $L \geq 1$.

1. Introduction

Interconnection networks play a central role in determining the overall performance of a multiprocessor/multi-computer system. If the networks cannot provide adequate performance for a particular application, nodes will frequently be forced to wait for data to arrive. Therefore, it is important to design an interconnection network with better performance for those properties. For this, let processes or cores be considered as vertices, and communication channels be considered as edges (or say links), thus all problems on interconnection networks can be discussed on graphs.

Suppose that G is a graph. A path (cycle) in G is called a *Hamiltonian path* (*Hamiltonian cycle*) if it contains every node of G exactly once. G is called *Hamiltonian* if there is a Hamiltonian cycle in G, and it is called *Hamiltonian-connected* if there is a Hamiltonian path between every two distinct nodes of G [1]. Some topologies, such as the hierarchical cubic network [2], are Hamiltonian-connected. A graph G is called *k-node Hamiltonian* if it remains Hamiltonian after removing any k nodes; a graph G is called *k-edge* (or say *k-link*) *Hamiltonian* if it remains Hamiltonian after removing any k edges (links) [3]. The reader is referred to [4] for those terms not defined here. In recent years, many researchers study Hamiltonian-connected on several topologies with faulty nodes or edges, like Cartesian product graphs [5,6], locally twisted cubes [7], augmented cubes [8,9], and so on.

A parallel algorithm is an algorithm which can be executed one piece at a time on many different processing devices, after which, the results are reassembled to obtain the correct result. This is useful because it has a great improvement on efficiency of the multiprocessing systems. If a network can embed the longest linear array between any two distinct nodes with dilation, congestion and load all equal to one, then all parallel algorithms designed for linear arrays can be

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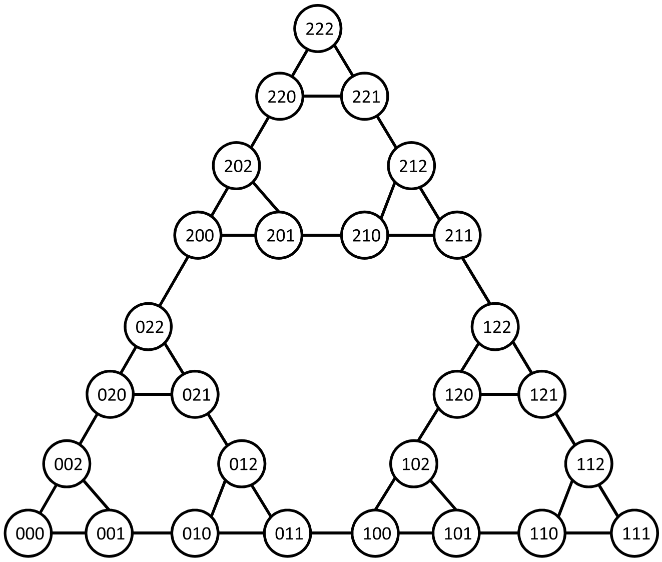

Fig. 1. Structure of $\mathrm{WK}_{(3,3)}$.

executed on this network as well. Therefore, the Hamiltonian properties of graphs are worth discussing when considering the performance of a network.

In 1986, Lu [10] proposed Hanoi graphs, which correspond to the allowed moves in the Tower of Hanoi problem. Hanoi graphs, triangular meshes, meshes and toroidal meshes are all Hamiltonian, but consider the same dimensions, although Hanoi graphs have more vertices than triangular meshes, they have fewer vertices than meshes and toroidal meshes. At the same time, Hanoi graphs have a lower degree for any internal vertex than each internal vertex in triangular meshes, toroidal meshes or meshes. This is a good property when making the structure into a physical hardware application. We can say that Hanoi graphs have better performance in some way than triangular meshes, toroidal meshes and meshes. Hence, Hanoi graphs have been interesting to some researchers in recent years, such as [11] and [12].

The d dimension t amplitude WK-recursive network for $d \geq 1, t \geq 1$ ($\mathrm{WK}_{(d,t)}$, for short), proposed in 1992 by Fernandes [13], is a network that is recursively defined and is expandable to any level. The $\mathrm{WK}_{(d,t)}$ is a general version of Hanoi graphs in fact, i.e. the Hanoi graph is isomorphic to $\mathrm{WK}_{(3,t)}$. There also have been several studies on WK-recursive networks ([14–16]). In 2005, Fu showed that a $\mathrm{WK}_{(d,t)}$ Network is Hamiltonian-connected and $(d-3)$-node Hamiltonian, for $d \geq 4$ and $t \geq 1$.

In 1993, Fernandes and Kanevsky proposed two hierarchical WK-recursive topologies, Hierarchical WK-Recursive networks and WK-recursive Pyramid networks for $d \geq 1, t \geq 1, L \geq 1$ ($\mathrm{HWK}_{(d,t,L)}$ and $\mathrm{WKP}_{(d,t,L)}$, for short, respectively) [17]. They are constructed by taking different sizes WK-recursive networks as different layers. Another work [17] discussed the number of orders and connectivity of $\mathrm{HWK}_{(d,t,L)}$ and $\mathrm{WKP}_{(d,t,L)}$. In 2008, Farahabady et. al. claimed that a special graph, called WKP, is Hamiltonian and pancyclic, and also proved those properties [18]. In fact, WKP is a special case of $\mathrm{WKP}_{(d,t,L)}$ in which $d=4$ and $t=1$.

In this paper, we discuss the more general special version of $\mathrm{WKP}_{(d,t,L)}$ in which $t=1$, denoted by $\mathrm{WKP}_{(d,L)}$ for $d \geq 1$ and $L \geq 1$. In $\mathrm{WKP}_{(d,L)}$, each vertex has exactly d children and the tth layer is isomorphic to a $\mathrm{WK}_{(d,t)}$. Therefore, the topology proposed by Razavi and Sarbazi-Azad in 2008 [16] is $\mathrm{WKP}_{(4,L)}$ by our definition.

The rest of this paper is organized as follows. Section 2 gives some definitions and show some preliminaries. Section 3 proves one of the main results, that $\mathrm{WKP}_{(d,L)}$ is Hamiltonian-connected for any positive integer $d \geq 3$, $L \geq 1$. Section 4 shows another main result that $\mathrm{WKP}_{(d,L)}$ is $(d-2)$-node Hamiltonian. Some conclusions are given in Section 5.

2. Definitions and preliminaries

In this section, we give the definition of $\mathrm{WKP}_{(d,L)}$ and present some preliminaries of $\mathrm{WK}_{(d,t)}$ and $\mathrm{WKP}_{(d,L)}$. First, we review the definition of WK-recursive networks, and Fig. 1 is an illustration of $\mathrm{WK}_{(3,3)}$.

Definition 1. (See [13].) A radix-t WK-recursive network, denoted as $\mathrm{WK}_{(d,t)}$, consists of a set of nodes $V(\mathrm{WK}_{(d,t)}) = \{a_{t-1}a_{t-2}...a_1a_0 | 0 \leq a_i \leq d, 0 \leq i \leq t-1\}$. Each node $a_{t-1}a_{t-2}...a_1a_0$ is adjacent to 1) $a_{t-1}a_{t-2}\ldots a_1 b$, where $b \neq a_0$; and 2) $a_{t-1}a_{t-2}\ldots a_{j+1}a_{j-1}(a_j)^j$, if $a_j \neq a_{j-1}$ and $a_{j-1} = a_{j-2} = \ldots = a_0$, where $(a_j)^j$ denotes j consecutive a_js.

For convenience, $a_{t-1}a_{t-2}...a_1a_0$ is called the *labeling* of a node in $V(\mathrm{WK}_{(d,t)})$. A node $a_{t-1}a_{t-2}...a_1a_0$ is called a *k-frontier* if $a_{k-1} = a_{k-2} = \ldots = a_0$, where $1 \leq k \leq t$. A k-frontier is *proper* if it is not a $(k+1)$-frontier.