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On the conditional diagnosability of matching composition networks

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ABSTRACT

The conditional diagnosability of interconnection networks has been studied by using a number of ad-hoc methods. Recently, gathering various ad-hoc methods developed in the last decade, a unified approach was developed, and this approach was used to find the conditional diagnosability of many interconnection networks. In this paper, we study the conditional diagnosability of matching composition networks, including those that are not triangle-free.

1. Introduction

Thanks to constant technological progress, multiprocessor systems with ever increasing number of interconnected computing nodes are becoming a reality. To address the reliability concern of such a system, it is ideal, and technically feasible, to have a self-diagnosable system where the computing nodes are able to detect faulty ones by themselves in the form of a *diagnosis*. One major approach to this regard is called the comparison diagnosis model [20,21], where each node performs a diagnosis by sending the same input to all pairs of its distinct neighbors and then comparing their responses. Based on such comparison results made by all the processors, the faulty status of the system can be decided. The number of detectable faulty nodes in such a multiprocessor system certainly depends on the topology of its associated interprocessor structure, as well as the modeling assumptions, and the maximum number of detectable faulty nodes in such a network is called its *diagnosability*. Such a measurement directly characterizes the fault-tolerance ability of an interconnection network and is thus of great interest [10,11,18,19,23,32].

When all the neighbors of some processor in a network are faulty simultaneously, it is impossible to determine the faulty status of this processor, as well as that of the whole system. Hence, the unrestricted diagnosability of a network, when represented with a graph G, is limited by the minimum degree of G, often too small thus unsatisfying. On the other hand, with the often made statistical assumption of independent and identical distribution (*i.i.d.*) of failures among processors, it is simply unlikely that all the neighbors of a certain processor will fail at the same time, hence the notion of *conditional diagnosability* was introduced in [18] which assumes that no *conditional faulty set* contains all the neighbors of any processor.

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This more realistic notion leads to an improved characterization of a network's fault-tolerance properties, and has since been identified for several networks, including hypercubes [16], k-ary n-cubes [13], folded hypercubes [14,24], augmented cubes [9,24], Cayley graphs generated by transposition trees [19], alternating group networks [32], BC Networks [15], pancake graphs [24], and (n,k)-star graphs [8], all under the above comparison diagnosis model. Much work has also been done under another diagnostic model, the PMC model [22] (with recent papers [3,4,12,33]), where diagnosis is made by testing adjacent nodes. Although it is pointed out in [23] that the comparison model generalizes the PMC model, diagnosability results achieved under the comparison model are often as large as those achieved under the PMC model. Initially, ad-hoc methods were used in determining the conditional diagnosability as in [18,19,31–33]. This topic has been slowly converging to a general scheme as developed in [8,9], and further solidified in [7].

The class of *bijective connection networks (BC networks)* is defined recursively as follows: Let $\mathcal{H}_1 = \{K_2\}$ and for $i \geq 2$, let $\mathcal{H}_i$ be the set of all graphs that can be constructed by taking two (possibly the same) elements $H_1 = (V_1, E_1)$ and $H_2 = (V_2, E_2)$ from $\mathcal{H}_{i-1}$ (if we take the same element, we will assume they are two different copies and so $V_1 \cap V_2$ remains empty) with a bijection $f : V_1 \longrightarrow V_2$ to form the graph $H = (V_1 \cup V_2, E_1 \cup E_2 \cup M)$ where $M = \{(v, f(v)) : v \in V_1\}$. This class of networks include a number of networks such as hypercubes, crossed cubes and twisted cubes. We note that BC networks are triangle-free, that is, they do not contain a K_3 as a subgraph. The class of *matching composition networks (MC networks or simply MCN's)* is defined as follows: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs with $|V_1| = |V_2|$, $f : V_1 \longrightarrow V_2$ be a bijection and $M = \{(v, f(v)) : v \in V_1\}$; then construct the graph $G = (V_1 \cup V_2, E_1 \cup E_2 \cup M)$. We remark that this is called matching composition network as M is a perfect matching in G. Often G is denoted by $G(G_1, G_2, M)$. In [15], the conditional diagnosability of BC networks was studied and recently the corresponding problem for triangle-free MCN's was studied in [25]. Since interconnection networks are usually regular, typically both G_1 and G_2 are set to be r-regular even though the definition allows more flexibility. In this paper, we consider several non-triangle-free families of MCN's. We remark that if one wants to build a class of MCN's that contain triangles, a natural definition would be the class of *triangle bijective connection networks (TBC networks)* and it is defined recursively as follows: Let $\mathcal{G}_3 = \{K_4\}$ and for $r \geq 4$, let $\mathcal{G}_r$ be the set of all graphs that can be constructed by taking two (possibly the same) elements $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ from $\mathcal{G}_{r-1}$ (if we take the same element, we will assume they are two different copies and so $V_1 \cap V_2$ remains empty) with a bijection $f : V_1 \longrightarrow V_2$ to form the graph $G = (V_1 \cup V_2, E_1 \cup E_2 \cup M)$ where $M = \{(v, f(v)) : v \in V_1\}$. So a TBC network in $\mathcal{G}_r$ is r-regular with 2^{r-1} vertices where $r \geq 3$.

Our main goal is to develop a general diagnosability result for matching composition networks. As such, there are a number of ideas that need to be utilized. In order to make the exposition clearer, we choose to accomplish this in several steps. We first prove a special case and then we successively generalize it. Although this makes the paper slightly longer, we believe that this is a better way to convey the crux of the concepts. The paper is organized as follows: Section 2 sets up the definitions and a basic result for TBC networks; Section 3 gives several known results that are our building blocks; Section 4 gives a general result and Section 5 uses this result to determine the conditional diagnosability of TBC networks. Section 6 and Section 7 give variants and generalizations.

2. Fundamental notions and results

In this paper, we follow the usual graph theory terminology. In particular, let G be a graph and $v \in V(G)$, $N_G(v)$ is the set of all the vertices adjacent to v and $N_G(S) = \bigcup_{v \in S} N(v) \setminus S$ where $S \subset V(G)$. We use $\delta(G)$ (respectively, $\Delta(G)$) to denote the minimum (respectively, the maximum) degree of vertices in G.

As mentioned earlier, according to the comparison diagnosis model, a vertex $w \in G$, a *comparator,* sends an input to every pair of neighbors v and x, and generates a result $r((v,x)_w)$, which equals 0 if both v and x send back the same response (and if w is not faulty). Clearly, if $r((v,x)_w) = 1$, then at least one of the three vertices is faulty. If w is faulty, then the result is unreliable. The collection of all such results is called the *syndrome of the diagnosis.* (For this model to function properly, there are a number of assumptions. We refer the readers to [9] for details.) A subset $F \subset V(G)$ is said to be *compatible with a syndrome* r if r can be generated when all vertices in F are faulty and those in $V(G) \setminus F$ are fault-free. Finally, a graph $G(V,E)$ is *diagnosable* if, for every syndrome r, there is a unique $F \subset V(G)$ compatible with r. One of the related problems is certainly, given such a syndrome of a network, how to identify its associated compatible faulty set. Various algorithms have been designed to serve this purpose under both the comparison diagnosis and the PMC models in [23,29,33].

On the other hand, [23] pointed out that two faulty sets may be compatible with the same syndrome. Such an observation leads to the notion of a *t-diagnosable graph* [23]: a graph is *t-diagnosable* as long as the size of the aforementioned unique faulty set F is no more than t. In this context, the *diagnosability of a graph* G, denoted as $t(G)$, is defined to be the maximum number of faulty vertices that G can guarantee to identify, and the *conditional diagnosability of* G, denoted as $t_c(G)$, is defined to be the maximum number of faulty vertices that G can guarantee to identify, when no faulty set includes all the neighbors of any vertex in G. More specifically, two distinct faulty sets F_1 and F_2 are *indistinguishable* if and only if they are compatible with at least one syndrome, *distinguishable* otherwise. Hence, $t(G)$ equals the maximum number q such that for all distinct faulty set pairs (F_1, F_2), $|F_1| \leq q$, $|F_2| \leq q$, F_1 and F_2 are distinguishable. We have a similar specification for $t_c(G)$.