Theoretical Computer Science 539 (2014) 1–12

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Revenue maximization in a Bayesian double auction market

Xiaotie Deng [a,1], Paul Goldberg [b,2], Bo Tang [c,*], Jinshan Zhang [c,*]

[a] Department of Computer Science, Shanghai Jiao Tong University, China
[b] Department of Computer Science, University of Oxford, United Kingdom
[c] Department of Computer Science, University of Liverpool, United Kingdom

ARTICLE INFO

Article history:
Received 15 May 2013
Received in revised form 29 January 2014
Accepted 12 April 2014
Communicated by K.-M. Chao

Keywords:
Bayesian setting
Double auction
Revenue maximization
Optimal auction
Approximate auction

ABSTRACT

We study double auction market design where the market maker wants to maximize its total revenue by buying low from the sellers and selling high to the buyers. We consider a Bayesian setting where buyers and sellers have independent probability distributions on the values of products on the market.

For the simplest setting, each seller has one kind of indivisible good with a bounded (integer) amount that can be sold to a buyer, who may demand a bounded number of copies. We develop a maximum mechanism for the market maker to maximize its own revenue.

For the more general case where each seller's product may be different, we consider a number of variants in terms of constraints on supplies and demands. For each of them, we develop a polynomial time computable truthful mechanism for the market maker to achieve a revenue at least a constant α times the revenue of any other truthful mechanism.

1. Introduction

We consider a double auction market maker who collects valuations from buyers and sellers about a certain product to decide on the prices each seller gets and each buyer pays. The buyers may want to buy many units and the sellers may have many units to part with. The buyers and sellers may have different valuations of the product, and the probability distributions of the valuations are public knowledge but each valuation, sampled from its distribution, is known only to its own buyer or seller. For simplicity, we assume that the probability distributions are independent. For the sellers and buyers, they know their own private values exactly. The market maker purchases the products from the sellers and sells them to the buyers. Our goal is to design a market mechanism that maximizes the revenue of the market maker. In other words, the market maker is to buy the same amount of products from the sellers as the amount sold to the buyers with the objective of maximizing the difference of its collected payment from the buyers and the total amount paid to the sellers. When in addition we assume public knowledge of distributions of buyers' private values from the previous sales, we call it a revenue maximization Bayesian double auction market maker.

* Corresponding author.
E-mail addresses: dengxiaotie@gmail.com (X. Deng), Paul.Goldberg@cs.ox.ac.uk (P. Goldberg), Bo.Tang@liverpool.ac.uk (B. Tang), Jinshan.Zhang@liverpool.ac.uk (J. Zhang).

[1] Supported by the National Science Foundation of China (Grant No. 61173011) and a Project 985 grant of Shanghai Jiao Tong University.
[2] Supported by EPSRC grant EP/G069239/1 "Efficient Decentralised Approaches in Algorithmic Game Theory".

http://dx.doi.org/10.1016/j.tcs.2014.04.013
0304-3975/

Table 1
Results.

	Dimension	Demand	Supply	Distribution	Results
Section 3	Single	Arbitrary	Arbitrary	Continuous/Discrete	Optimal
Section 4	Multi	Arbitrary	Arbitrary	Continuous	1/4-Approx
Section 4	Multi	Arbitrary	Arbitrary	Discrete	1/4-Approx
Section 5	Multi	Unlimited	Arbitrary	Discrete	Optimal
Section 5	Multi	Arbitrary	Unlimited	Discrete	Optimal

For the demand column, "Arbitrary" refers to the case where buyers can buy at most d_i items where d_i can be an arbitrary number and "Unlimited" means $d_i = +\infty$. The supply column is similar.

There have been many double auction institutions, each of which may be suitable for one type of market environment [9]. Ours is motivated by the growing use of discriminative pricing models over the Internet such as one that is studied in [7] for the prior-free market environment. A possible realistic setting for applications of our model could be Google's ad exchange where Google could play a market maker for advertisers and webpage owners [13]. One may also use it for a market model of Groupon. Our use of the Bayesian model is justified by the repeated uses of a commercial system by registered users. It allows the market maker to gain Bayesian information of the users' valuations of the products being sold. Therefore, the Bayesian model adequately describes the knowledge of the market maker, buyers and sellers for the optimal mechanism design.

Our results. We provide optimal or constant approximate mechanisms for various settings for double auction design. The parameters considered in our discussion are related to important market design issues. Those include one or multi dimensional problems (meaning, one product or multiple different types of products). The buyers can have demand constraints or not. The sellers can be supply constrained or not. Players' values may be drawn from a continuous or from a discrete distribution. The results are summarized in Table 1. In the Bayesian Mechanism Design problems, there are two computational processes involved. The first one is to design an optimal or approximate mechanism which can be viewed as a function mapping bidders' profiles to allocation and payment outcomes. Since the function maps potentially exponentially many profiles to outcomes, a succinct representation of the function is an important part in the Bayesian mechanism design. The second process is the implementation of the mechanism, i.e., given a bid profile, we run the mechanism to compute the outcome allocation and payment scheme. Our results imply that all mechanisms described in the table can be represented in polynomial size and can be found and implemented in polynomial time.

Related works. Auction design plays an important role in economics in general and especially in electronic commerce [12]. Of particular interest, is the problem of maximizing the auctioneer's revenue, referred as the optimal auction design problem. A number of research works have focused on this issue. Myerson, in his seminal paper [14], characterized the optimal auction for the single-item setting in the Bayesian model. Recently, efforts have been made on extending Myerson's results to border settings [8,16,18].

Unlike Myerson's optimal auction result, finding the optimal solution is not easy for multi-dimensional settings. Recent research interest has turned toward approximate mechanisms [1,5]. Cai et al. [4] presented a characterization of a rather general multi-dimensional setting and proposed an efficient mechanism for the special case where no bidders are demand constrained. Using similar ideas, Alaei et al. [2] present a general framework for reducing multi-agent service problems to single-agent ones.

The double auction design problem becomes more complicated since the market maker acts as the middle man to bring buyers and sellers together. A guide to the literature in micro-economics on this topic can be found in [9]. The profit maximization problem for the single buyer/single seller setting has been studied by Myerson and Satterthwaite [15]. Our optimal double auction is a direct extension of their work and, to our best knowledge, fills a clear gap in the economic theory of double auctions. Deshmukh et al. [7], studied the revenue maximization problem for double auctions where the auctioneer has no prior knowledge about bids. Their prior-free model is essentially different from ours. More auction mechanism design problems were studied by many researchers in recent years, but as far as we know, not in the context of optimal double auction design in the Bayesian setting. The most related one is by Jain and Wilkens [10], where they studied the market intermediation problem in a setting with a single unit-demand buyer and a group of sellers. They gave several constant approximate mechanisms with various buyer behavior assumptions. While our setting assumes the existence of a monopoly platform, Rochet and Tirole [17] and Armstrong [3] introduced several different models for the two-sided market and studied the platform competition problem.

2. Preliminaries

Throughout the paper we focus on Bayesian incentive compatible mechanisms only. Informally, a mechanism is Bayesian incentive compatible if it is optimal (in the expected utility) for each buyer and each seller to bid its true value of the items. We will formally define this concept later. As a consequence, we should consider their bids to be their true valuations and