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A linear time algorithm for embedding hypercube into cylinder and torus

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ABSTRACT

In this paper we solve two conjectures proposed by Manuel et al. (2011) [7] to obtain exact wirelength of embedding an r-dimensional hypercube into cylinder $C_{2^{r_1}} \times P_{2^{r_2}}$ and torus $C_{2^{r_1}} \times C_{2^{r_2}}$, where $r_1 + r_2 = r$ and $r_1 \leq r_2$. We provide a linear time algorithm to compute the exact wirelength of embedding hypercube into cylinder and torus. Further we extend the result for higher dimensional cylinder and torus.

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1. Introduction

In recent years, among many interconnection networks, the hypercube has been the focus of many researchers due to its structural regularity, potential for parallel computation of various algorithms, and the high degree of fault tolerance [1]. Hypercubes are known to simulate other structures such as grids and binary trees [2,3].

Graph embedding is an important technique that maps a logical graph into a host graph, usually an interconnection network. Many applications can be modeled as graph embedding [4–8]. The quality of an embedding can be measured by certain cost criteria. One of these criteria is the *wirelength*. The wirelength of a graph embedding arises from VLSI designs, data structures and data representations, networks for parallel computer systems, biological models that deal with cloning and visual stimuli, parallel architecture, structural engineering and so on [6,9].

Graph embeddings have been well studied for hypercubes into grids [2], meshes into crossed cubes [10], meshes into locally twisted cubes [11], meshes into faulty crossed cubes [12], generalized ladders into hypercubes [13], rectangular grids into hypercubes [14,15], grids into grids [16], binary trees into grids [17], meshes into Möbius cubes [18], tori and grids into twisted cubes [19], hypercube into n-dimensional grid [20].

Even though there are numerous results and discussions on the wirelength problem, most of them deal with only approximate results and the estimation of lower bounds [21,22]. But the Congestion Lemma and the Partition Lemma

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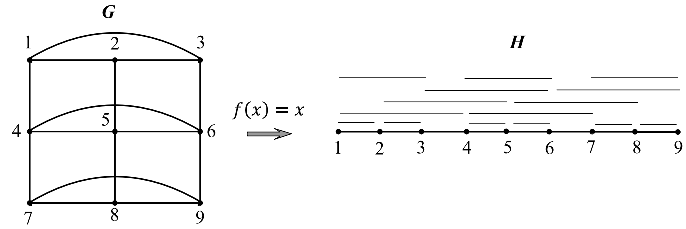

Fig. 1. Wiring diagram of a cylinder G into path H with $WL_f(G, H) = 30$.

[2,7] have enabled the computation of exact wirelength for embeddings of various architectures [2,7,10,11,20,23]. In fact the technique focuses on specific partitioning of the edge set of the host graph. It is interesting to note that not all host graphs can be partitioned to apply the Partition Lemma. In this paper, we overcome this difficulty partially by retaining a set of edges on which minimum wirelength is computed using Partition Lemma and compute minimum congestion on the rest of the edges using various other procedures.

Definition 1.1. (See [21].) Let G and H be finite graphs. An *embedding* f of G into H is defined as follows:

1. f is a one-to-one map from $V(G) \to V(H)$
2. P_f is a one-to-one map from $E(G)$ to $\{P_f(u, v) : P_f(u, v)$ is a path in H between $f(u)$ and $f(v)$ for $(u, v) \in E(G)\}$.

The *expansion* of an embedding f is the ratio of the number of vertices of H to the number of vertices of G. In this paper, we consider embeddings with expansion one.

Definition 1.2. (See [21].) The *edge congestion* of an embedding f of G into H is the maximum number of edges of the graph G that are embedded on any single edge of H. Let $EC_f(e)$ denote the number of edges (u, v) of G such that e is in the path $P_f(u, v)$ between the vertices $f(u)$ and $f(v)$ in H. In other words,

$$EC_f(e) = \left|\left\{(u, v) \in E(G) : e \in P_f(u, v)\right\}\right|$$

where $P_f(u, v)$ denotes the path between $f(u)$ and $f(v)$ in H with respect to f.

If we think of G as representing the wiring diagram of an electronic circuit, with the vertices representing components and the edges representing wires connecting them, then the edge congestion $EC(G, H)$ is the minimum, over all embeddings $f : V(G) \to V(H)$, of the maximum number of wires that cross any edge of H [24]. See Fig. 1.

Definition 1.3. (See [2].) The wirelength of an embedding f of G into H is given by

$$WL_f(G, H) = \sum_{(u,v)\in E(G)} \left|P_f(u, v)\right| = \sum_{e\in E(H)} EC_f(e)$$

where $|P_f(u, v)|$ denotes the length of the path $P_f(u, v)$ in H.

The *wirelength* of G into H is defined as

$$WL(G, H) = \min WL_f(G, H)$$

where the minimum is taken over all embeddings f of G into H.

The *wirelength problem* [2,17,21,22,24] of a graph G into H is to find an embedding of G into H that induces the minimum wirelength $WL(G, H)$. The isoperimetric problem [25] has been used as a powerful tool in the computation of exact wirelength of graph embeddings. The problem is to determine a subset A of vertices of a graph G such that $\theta_G(A) = \theta_G(m)$, where $\theta_G(A) = \{(u, v) \in E : u \in A, v \notin A\}$ and for a given m, $\theta_G(m) = \min_{B\subseteq V,|B|=m} |\theta_G(B)|$. Such subsets are called optimal [25,28].

The maximum subgraph problem [25] is to find a subset of vertices of a given graph, such that the number of edges in the subgraph induced by this subset is maximal among all induced subgraphs with the same number of vertices. Mathematically, for a given m, if $I_G(m) = \max_{A\subseteq V,\ |A|=m} |I_G(A)|$ where $I_G(A) = \{(u, v) \in E : u, v \in A\}$, then the problem is to find $A \subseteq V$ such that $|A| = m$ and $I_G(m) = |I_G(A)|$. The maximum subgraph problem is NP-complete [26]. When G is regular, the isoperimetric problem is equivalent to the maximum subgraph problem.

Lemma 1.4 *(Congestion Lemma). (See [2,7].) Let G be an r-regular graph and f be an embedding of G into H. Let S be an edge cut of H such that the removal of edges of S leaves H into 2 components H_1 and H_2 and let $G_1 = f^{-1}(H_1)$ and $G_2 = f^{-1}(H_2)$. Also S satisfies the following conditions:*

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