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Conditional diagnosability of optical multi-mesh hypercube networks under the comparison diagnosis model

Xianyong Li [a,*], Xiaofan Yang [a,b], Li He [a,c], Jing Zhang [a], Cui Yu [a]

[a] College of Computer Science, Chongqing University, Chongqing 400044, PR China
[b] School of Electronic and Information Engineering, Southwest University, Chongqing, 400715, PR China
[c] College of Computer Science, Chongqing University of Posts and Telecommunications, Chongqing 400065, PR China

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ABSTRACT

Due to integrated positive features of both hypercubes and tori, optical multi-mesh hypercube (OMMH) networks are regarded as a class of promising optical interconnection topologies. The notion of conditional diagnosability helps enhance the self-diagnosing capability of multicomputers. This paper determines the conditional diagnosabilities of OMMH networks under the Maeng–Malek comparison model.

1. Introduction

With the ever increasing size of multicomputers, the possibility that faulty processors (nodes) are present in such systems is becoming increasingly large. The so-called *system-level diagnosis*, which aims at automatically identifying the faulty nodes present in a system, is widely regarded as an effective means of maintaining its high availability [28]. See Ref. [6] for a comprehensive review about this subject. Recently, system-level diagnosis technique has been applied successfully to mobile ad hoc networks [5,7] and optical networks [33,34].

The conventional fault diagnosis is conducted provided that any set of nodes may fail simultaneously, leading to the relatively conservative result that the diagnosability of such a system cannot exceed the minimum vertex degree of its underlying network. In practical situations, however, the probability that all neighbors of some vertex in the network fail simultaneously is vanishingly small and, hence, can be ignored. Taking this fact into account, Lai et al. [20] introduced the notion of *conditional diagnosability*, significantly enhancing the self-diagnosing capability of multicomputers. In this context, a crop of interesting results have been obtained. Roughly speaking, under the comparison diagnosis model [26,27,29], the conditional diagnosabilities of an n-dimensional cube, an n-dimensional augmented cube, and an n-dimensional BC network are $3n-5$, $6n-17$, and $3n-5$, respectively [15,9,16], whereas their counterparts under the classical PMC model are $4n-7$, $8n-27$, and $4n-7$, respectively [20,3,38]. For more information on this issue, see Refs. [13,14,21,32,35–37]. Finally, it is worth mentioning that Hsieh et al. have explored other kinds of diagnosabilities [4,10–12,18,19]. Due to appealing properties, ranging from extremely high bandwidth to extremely low power consumption and extremely low latency, optical interconnection networks have emerged as a promising alternative to electrical interconnection networks [8,30]. Recently,

* Corresponding author.
E-mail addresses: xian-yong@163.com (X. Li), xfyang1964@gmail.com (X. Yang).

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the conditional diagnosabilities of the optical hypermesh networks were derived under the PMC and comparison models, respectively [33,34].

In 1994, Louri and Sung [24,25] suggested a class of promising optical interconnection topologies, known as the *optical multi-mesh hypercube* (OMMH) networks, which possess a two-level structure: a local connection level representing a collection of cube modules and a global connection level representing a torus network connecting the hypercube modules. Typically, an OMMH network can be characterized by a triplet (l, m, n), where l and m represent the numbers of rows and columns of the torus, respectively, n the dimension of the cube module. OMMH networks integrate positive features of both hypercubes (smaller diameter, larger connectivity, excellent symmetry and fault tolerance, simpler routing strategies) and tori (constant node degree and excellent scalability). Indeed, OMMH networks have been physically demonstrated using a combination of free-space and optical fiber technologies, showing good performance characteristics [22,23]. To our knowledge, however, the conditional diagnosability of OMMHs under the Maeng–Malek comparison model has yet to be determined.

This paper investigates the conditional diagnosabilities of OMMH networks under the Maeng–Malek comparison model. For that purpose, some interesting properties of OMMH networks are revealed. On this basis, the conditional diagnosability of an (l, m, n)-OMMH network is shown to be $3n + 7$ if either (a) $n \geqslant 1$, $l, m \geqslant 4$, or (b) $n = 0$, $l, m \geqslant 4$, $(l, m) \neq (4, 4), (4, 5), (5, 4), (4, 6), (6, 4)$.

This paper is organized as follows: Section 2 introduces preliminary knowledge. Section 3 gives some interesting properties of OMMH networks. Section 4 is devoted to the proof of the main result. This work is closed by Section 5.

2. Preliminaries

For a vertex subset S of graph G, let $N_G(S)$ denote its *neighborhood*, i.e.,

$$N_G(S) = \{u \in V(G) \setminus S\colon\ u \text{ is adjacent to some vertex in } S\}.$$

For a graph G, let $\kappa(G)$, $\delta(G)$, $\alpha(G)$ and $\beta(G)$ denote its connectivity, its minimum vertex degree, its independence number and its matching number, respectively. For other fundamental graph-theoretic notations and terminologies, see Ref. [2].

For our purpose, a multicomputer system shall be represented by its underlying interconnection network, an undirected graph $G = (V, E)$ whose vertices and edges stand for processors and communication links between processors, respectively. A *fault set* of a graph is a vertex subset that may possibly be the set of all faulty vertices present in the graph. A fault set is *t-fault* if it contains no more than t vertices. A fault set is *conditional* if it does not include the neighborhood of any vertex. A fault set is *conditional t-fault* if it is both conditional and t-fault.

Under the Maeng–Malek comparison model (MM model, for short), the *comparison graph* for a graph $G = (V, E)$ is defined as an edge-labeled multigraph $M^* = (V, C)$, where C contains an edge (u, v) labeled with w, denoted $(u, v)_w$, if and only if vertices u and v are both adjacent to vertex w. Let $\sigma((u, v)_w) = 0$ denote that w judges that the outputs produced by u and v are identical, and let $\sigma((u, v)_w) = 1$ denote that w judges that the outputs produced by u and v are different. Suppose vertex w is fault-free, then $\sigma((u, v)_w) = 0$ implies that u and v are both fault-free, whereas $\sigma((u, v)_w) = 1$ implies that at least one of the three vertices in question is faulty. All comparison results, denoted $\sigma : C \to \{0, 1\}$, form a *syndrome*.

A syndrome σ is *consistent* with a fault set F if σ can be produced by F. Let

$$\sigma(F) = \{\sigma\colon\ \sigma \text{ is consistent with } F\}.$$

Two distinct fault sets $F_1, F_2 \subseteq V$ are *distinguishable* if $\sigma(F_1) \cap \sigma(F_2) = \emptyset$, otherwise they are *indistinguishable*.

Definition 1. (See [20].) A graph is *conditionally t-diagnosable* if any two distinct conditional t-fault sets in the graph are distinguishable. The *conditional diagnosability* of a graph G, denoted $t_c(G)$, is defined as the maximum integer t such that G is conditionally t-diagnosable.

For a pair of sets, S_1 and S_2, let $S_1 \triangle S_2 = (S_1 \setminus S_2) \cup (S_2 \setminus S_1)$.

Lemma 1. *(See [15,17,29].) Let F_1, F_2 be two distinct vertex subsets of graph G. Then, F_1 and F_2 are distinguishable if and only if either*

(C1) *$G \setminus (F_1 \cup F_2)$ has a vertex w that has a neighbor u in $G \setminus (F_1 \cup F_2)$ and has a neighbor v in $F_1 \triangle F_2$, or*
(C2) *there exist two vertices u, v both in $F_1 \setminus F_2$, or both in $F_2 \setminus F_1$, and there exists a vertex w in $V(G) \setminus (F_1 \cup F_2)$ such that $(u, v)_w \in C$.*

See Fig. 1 for an explanation of this lemma.

Definition 2. (See [24,25].) An (l, m, n)-OMMH network, denoted $MH_n^{l \times m}$, is defined as the Cartesian product graph $C_l \times C_m \times Q_n$, where C_r denotes an r-cycle whose vertices are labeled sequentially as $0, 1, \ldots, r - 1$, Q_n denotes an n-cube.